Heterogeneous Workers and Occupations: Inequality, Unemployment, and Crowding Out

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This article attempts to determine the factors behind the cyclical behavior of the skill premium. Using the Current Population Survey Outgoing Rotation Group, the premium is found uncorrelated with contemporaneous output and lags the business cycle. To account for this observation, the article develops a framework that features search frictions. Agents are either high or low educated, and firms post two types of vacancies: the complex, which can be matched with the high educated, and the simple, which can be matched with the high and the low educated. On-the-job search for a complex occupation is undertaken by the high educated in simple occupations. An aggregate technological shock induces firms to increase their posting of simple and complex vacancies. As complex vacancies are more costly to create than simple ones, the increase in the former lags that of the latter. As the number of complex vacancies increases with a lag, on-the-job search intensity increases on the expense of the hours of work of the high educated in simple occupations. On the other hand, the total hours of the high educated in complex occupations increases. This lagged cyclical increase in the labor input of the high educated in complex occupations causes the skill premium to increase with a lag as well.

1. Introduction

This article attempts to determine the factors behind the cyclical behavior of the skill premium. Previous studies attempted to explain the observation that the skill premium is uncorrelated with contemporaneous output and lags the business cycle. However, these studies encountered several problems that rendered this observation a puzzle remaining to be properly addressed. To this purpose, the article derives a set of stylized facts that demonstrate not only the cyclical behavior of the skill premium but also that of other variables that capture the cyclical allocation of labor input in a labor market with heterogeneous agents across educational levels. These additional observations reflect a lagged cyclical upgrading of jobs by the college educated, or a lagged cyclical increase in their labor input from jobs that do not require college education to ones that do. This causes the gap between the wage of the high educated and that of the low educated to widen with a lag and accordingly can provide a possible explanation to the lagged procyclical skill premium. This intuition is used to develop a model that is capable of reproducing the observation of interest.

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Using the Outgoing Rotation Group of the Current Population Survey for the period from 1979 to 2004, the participants are divided into those who are employed and those who are unemployed. The two groups are further divided into those who are high and low educated, where the former are those with at least some college education. The employed types are further divided into those working in complex and in simple occupations, where the former are those jobs that require at least some college education. Therefore, a monthly data set is compiled including measures of the skill premium, the total hours of the high educated in complex and in simple occupations, the total hours of the low educated in simple occupations, the unemployment ratios of the high and the low educated, and a measure of the crowding out of the low educated by the high educated in occupying simple jobs. The observations suggest that the skill premium is uncorrelated with contemporaneous output and lags the business cycle. In addition, an economic expansion is accompanied contemporaneously by an increase in the total hours of all labor types employed in simple occupations, followed with a lag by an increase in the total hours of those employed in complex occupations and a decrease in the crowding-out effect. These observations reflect a possible lagged cyclical upgrading of jobs by the high educated through increasing their level of employment or their hours of work in complex occupations. This causes the gap between their wage and that of the low educated to widen and accordingly can explain the lagged procyclical skill premium.

The article develops a model to identify the underlying market interactions that are critical in generating the observed behavior along the lines of this intuition. These interactions are captured in a dynamic stochastic general equilibrium model that features search frictions. The households are divided into those high and low educated, and firms post two types of vacancies: the complex, which can be matched with the high educated, and the simple, which can be matched with both the high and the low educated. The high educated in simple occupations are allowed to search on the job for a complex occupation. An aggregate technological shock induces firms to increase their posting of simple and complex vacancies. As complex vacancies are more costly to create compared to simple ones, the increase in the former lags that of the latter. The lagged increase in the number of complex vacancies is accompanied by an increase in on-the-job search intensity. As the high educated in simple occupations increase the portion of time spent to search for a complex vacancy, their hours of work decline. On the other hand, because of the lagged increase in the employment and hours of work of the high educated in complex occupations, the total hours of this type increase. The wage of the high educated is a weighted average of the wage of those employed in complex and in simple occupations. The weights are given by the total hours in these occupations. Accordingly, these two factors cause the weight of the hours of work of the high educated in complex occupations to increase and the weighted average wage of the high educated to increase as well. This causes the gap between the wage of the high educated and that of the low educated to increase with a lag, causing the skill premium to increase with a lag as well.

This article adopts a different approach compared to previous studies that focused on the features of capital skill complementarity and variable utilization of capital to explain the cyclical pattern of the skill premium. For instance, Lindquist (2004) argued that capital skill complementarity allows the model to reproduce the cyclical pattern of the skill premium. However, Young (2003) argued that the success of the model in Lindquist (2004) is attributed to its abstraction from variable utilization of capital, whose introduction implies strong procyclical skill premium. Another criticism to the Lindquist (2004) framework is its inability to replicate the cyclical behavior of the underlying wages. Both Lindquist (2004) and Young
(2003) argued that the Walrasian aspects of the model cause the wages, which are equal to the marginal product of labor, to be strongly correlated with output. They suggested the introduction of implicit contracts to resolve this problem. Pourpourides (2007) developed a model with implicit contracts and demonstrated that the feature of variable utilization of capital, rather than capital skill complementarity, is essential in reproducing the observation that the skill premium is uncorrelated with contemporaneous output. The model, however, cannot explain the lagged procyclicality of the premium.

The remainder of the article is organized as follows: Section 2 presents the stylized facts, section 3 develops the model, section 4 includes the calibration, section 5 analyzes the results, and section 6 concludes. Data and derivations can be found in the Appendix.

2. Observations

To derive the business cycle patterns of selected labor market variables that reflect agent heterogeneity in educational levels and the educational requirements of jobs they are occupying, a time series is compiled from the Outgoing Rotation Group of the Current Population Survey. The survey provides monthly information from January 1979 until December 2004 on the participants’ employment status, level of education, type of occupation, weekly earnings, and weekly hours of work.

To compile a time series out of this survey, the labor market participants in each monthly file are divided into those employed and those unemployed. Each group is further divided into those high and low educated, where the former are those who obtained at least some college education. Each of the two employed groups is further divided into those working in a complex occupation and those working in a simple occupation, where the former is a job that requires at least some college education. This provides four employed and two unemployed types: the high educated employed in a complex occupation, the high educated employed in a simple occupation, the high educated unemployed, the low educated employed in a complex occupation, the low educated employed in a simple occupation, and the low educated unemployed. The low educated employed in a complex occupation are dropped from the sample because of their insignificant proportion out of all the low educated and out of all those employed in complex occupations. For the remainder of the employed types, weighted average hourly wages are calculated as the ratio of the weighted average weekly earnings to the weighted average weekly hours for each type. Using the hourly wages of the three employed types, the skill premium is defined as the ratio of the weighted average wage of the two high educated types to that of the low educated in simple occupations. Levels of employment are calculated for the three employed types, and levels of unemployment are calculated for the two unemployed types. Using the weighted average weekly hours of work of each group and the level of employment, the total hours of each group is derived. The proportion of each type out of the total sample is also calculated. Finally, a crowding-out variable is defined as the proportion of the total hours of the high educated among the total hours of all those employed in simple occupations, such that its increase reflects an increase in the crowding-out process of the low educated by the high educated in occupying this type of job.

Therefore, the variables compiled and used in the analysis are (i) the skill premium, (ii) the total hours of the high educated employed in complex occupations, (iii) the total hours of the

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1 Detailed data description is included in Appendix 1.
high educated employed in simple occupations, (iv) the total hours of the low educated employed in simple occupations, (v) the proportion of the high educated unemployed, (vi) the proportion of the low educated unemployed, and (vii) the crowding-out effect. This monthly time series are transformed into quarterly data by taking three month averages.

The cross-correlation coefficients between real gross domestic product (GDP) in period $t$ and each of these variables in lag and lead periods are displayed in Table 5. These patterns demonstrate that the skill premium has an insignificant correlation coefficient with contemporaneous output but is procyclical with a lag where the fourth lagged cross-correlation coefficient is statistically significant with a $p$-value of 0.0049. In addition, the total hours of the high educated in complex occupations is procyclical and lags the cycle by three quarters, as the cross-correlation coefficient with output reaches 0.4147, which is statistically significant with a $p$-value of zero. The proportion and the hours of the high and the low educated in simple occupations are procyclical. Therefore, the total hours of the high and the low educated employed in simple occupations are positively correlated with contemporaneous output with cross-correlation coefficients of 0.5317 and 0.7657, respectively, that are statistically significant with $p$-values of zero. The proportion of the high educated unemployed is countercyclical and lags the cycle where the cross-correlation coefficient with output reaches $-0.5243$ and is statistically significant, while the proportion of the low educated unemployed is countercyclical and lags the cycle where the cross-correlation coefficient with output of $-0.7230$ is also statistically significant.

Finally, the crowding-out effect is countercyclical with a lag as the fourth lagged cross-correlation coefficient of $-0.3335$ is statistically significant. These patterns are summarized as follows:

(i) The skill premium is procyclical with a lag.
(ii) The proportion of the high educated in complex occupations is procyclical with a lag.
(iii) The hours of the high educated in complex occupations is procyclical with a lead.
(iv) The total hours of the high educated in complex occupations is procyclical with a lag.
(v) The proportion of the high educated in simple occupations is procyclical.
(vi) The hours of the high educated in simple occupations is procyclical.
(vii) The total hours of the high educated in simple occupations is procyclical.
(viii) The proportion of the low educated in simple occupations is procyclical.
(ix) The hours of the low educated in simple occupations is procyclical.
(x) The total hours of the low educated in simple occupations is procyclical.
(xi) The proportion of the high educated unemployed is countercyclical with a lag.
(xii) The proportion of the low educated unemployed is countercyclical with a lag.
(xiii) The crowding-out effect is countercyclical with a lag.

The novel approach of this article is the use of the cyclical behavior of the variables pertaining to the allocation of labor input to ascertain intuitively the factors behind the business cycle pattern of the skill premium. For instance, the lagged increase in the total hours of the high educated in complex occupations reveals a possible lagged procyclical upgrading of jobs they are occupying. The procyclicality of the wage premium could be attributed to this process of job-to-job mobility by the high educated because as they move from simple to complex occupations or as they increase their hours of work in complex occupations, the gap between their wage and that of the low educated widens.

Few studies in the applied labor realm or in the business cycle literature attempted an investigation of the cyclical behavior of some of the variables studied in this article. The variable
that received the most attention is the skill premium. The premium was found countercyclical originally by Reder (1955), Azariadis (1976), and Kydland (1984). The dependence on aggregated data for the measurement of real wages, however, has been criticized, as it obscures the effect of compositional changes in the workforce over the cycle, as argued by Bils (1985) and Solon, Barsky, and Parker (1994). Therefore, some studies resorted to longitudinal data for the analysis of the cyclical pattern of the wage premium. In this context, Raisian (1983) used the Panel Study of Income Dynamics for the period 1967–1979 and found that skilled workers accept procyclical wage variability in exchange for more stable employment compared to unskilled workers, which induces a procyclical wage premium. Ziliak, Wilson, and Stone (1999) used the Panel Study of Income Dynamics for the period 1971–1990 and found that the rates of return to schooling are procyclical for whites but tend to be countercyclical for blacks. Keane and Prasad (1993) used the National Longitudinal Survey of Young Men for the period 1966–1981 and found the premium to be uncorrelated with contemporaneous measures of the business cycle.

Recent studies relied on compiled data from the Current Population Survey. In this context, Young (2003) found that during the period 1979–2001, the premium had significant volatility and a positive weak association with industrial production. Castro and Coen-Pirani (2008) found that the skill premium remained acyclical and not very volatile relative to GDP throughout the period from 1979 to 2003. Finally, Lindquist (2004) found that during the period 1979–2002, the mean skill premium was weakly positively correlated, while the median skill premium was uncorrelated with contemporaneous output, and that both lag output. These findings are consistent with the observation presented in this article that the skill premium is uncorrelated with contemporaneous output and exhibits lagged procyclicality.

Other studies compared the cyclical patterns of the employment and unemployment of the high and the low educated. For instance, Kydland (1984) used the Panel Study of Income Dynamics for the period 1970–1980 and concluded that unskilled workers in the United States experience greater employment fluctuations over the business cycle compared to skilled workers. Similarly, Hoynes (2000) used the Current Population Survey Outgoing Rotation Group for the period 1979–1992 and the March Annual Demographic files covering the period 1975–1997 and provided evidence that the employment rates of those with lower education exhibit more cyclical fluctuation and variability than those with higher education. Moreover, Keane, Moffitt, and Runkle (1988) used the National Longitudinal Survey of Young Men for the period 1966–1981 and found that unskilled workers are more likely to leave employment in a cyclical downturn. Farber (2005) used the Displaced Workers Survey over the period 1981–2003 and concluded that there is a strong cyclical pattern in job loss rates for unskilled workers compared to skilled workers. These results are also consistent with the evidence provided in this article as to the higher correlation coefficient with contemporaneous output for the unemployment ratio of the low educated compared to that of the high educated. Other studies analyzed the cyclical adjustment of jobs. For instance, Devereux (2000, 2004) used the Panel Study of Income Dynamics for the period 1976–1992 and found that in a recession the skilled occupy jobs that would normally be occupied by the unskilled.

3. Model

Consider an economy where time is infinite and discrete. The population is of measure 1, and there is a constant fraction δ of households that are ex ante high educated and (1 − δ) that
are low educated. The representative firm posts complex and simple vacancies. The complex vacancies are matched with the high educated only, while the simple vacancies are matched with both the high and the low educated. The firm also chooses the proportion of simple vacancies directed toward the high educated and that directed toward the low educated. A high educated worker in a simple occupation is allowed to continue searching on the job for a complex occupation. This is justified, as the two types of vacancies differ according to their creation costs, and these costs generate rents, which give rise to equilibrium wage differentials between occupation types. The model is an extension of Gautier (2002) in a general equilibrium framework and focuses on the dynamics of the model to explain some aspects of the business cycle.

Households

In this context, the high and low educated household members are divided into those employed and those unemployed as follows:

\[ N^hc_t + N^hs_t + U^h_t = \delta, \]  
\[ N^ct_t + U^c_t = 1 - \delta, \]

where \( N^j_t \) denotes the number of workers of education type \( i \) in occupation type \( j \), where \( i \in (h, l) \) for high and low educated workers respectively, and \( j \in (c, s) \) for complex and simple occupations, respectively. \( U^j_t \) denotes the number of the unemployed of type \( i \). Time for all types is normalized to one. A high educated unemployed uses a portion \( S^hc_t \) of his time to search for a complex occupation, a portion \( S^hs_t \) to search for a simple occupation, and \((1 - S^hc_t - S^hs_t)\) for leisure. A low educated unemployed uses a portion \( S^ct_t \) of his time to search for a simple occupation and \((1 - S^ct_t)\) for leisure. A high educated worker in a complex occupation spends a portion \( H^hc_t \) hours at work and \((1 - H^hc_t)\) for leisure. A high educated worker in a simple occupation spends a portion \( H^hs_t \) hours at work, a portion \( O_t \) to search on the job for a complex occupation, and \((1 - H^hc_t - O_t)\) for leisure. The low educated in a simple occupation spends a portion \( H^hs_t \) hours at work and \((1 - H^hs_t)\) for leisure.

As different employment histories among members of a household can lead to heterogeneous wealth positions, we follow the literature in assuming that each household is thought of as an extended family whose members perfectly insure each other against variations in labor income due to employment or unemployment. Remaining within the confines of complete markets allows solving the program of a representative household who chooses consumption and search intensities to maximize the expected discounted infinite sum of its instantaneous utility, which is separable in consumption and leisure. Assuming the household has the following value function, \( \Gamma^H_t = \Gamma^H_t \left( H^hc_t, N^hc_t, H^hs_t, N^hs_t, N^ct_t, O_t \right) \), the optimization problem of the household can be written in the following recursive form:

\[ \Gamma^H_t = \max_{\{C_t, S^hc_t, S^hs_t, O_t, S^ct_t\}} \left\{ \mathcal{U} (C_t) + U^h_t \mathcal{Q}^h_t + U^c_t \mathcal{Q}^c_t + N^hc_t \mathcal{Q}^hc_t + N^hs_t \mathcal{Q}^hs_t + N^ct_t \mathcal{Q}^ct_t + \beta E_t \left[ \Gamma^H_{t+1} \right] \right\}, \]

where \( E_t \) is the expectation operator conditional on the information set available in period \( t \), \( \beta \) is the discount factor, and \( \mathcal{U} (C_t) \) is the utility of period \( t \) consumption of the household \( C_t \).
\[ \Omega^h_t = \Omega^h (1 - S^h_t - S^v_t) \text{ and } \Omega^l_t = \Omega^l (1 - S^v_t) \] denote the utility of period \( t \) leisure of the high and low educated unemployed, respectively. \( \Omega^hc = \Omega^hc (1 - H^hc_t) \), \( \Omega^ls = \Omega^ls (1 - H^ls_t - O_t) \), and \( \Omega^l = \Omega^l (1 - H^l_t) \) denote the utility of period \( t \) leisure of the employed types. This is subject to the following budget constraint:

\[ C_t = N^hs_t H^hs_t W^hs_t + N^hs_t H^hs_t W^hs_t + N^hc_t H^hc_t W^hc_t + D_t, \] (4)

where \( W^hs_t \) is the period \( t \) wage for labor type \( ij \), and \( D_t \) is the dividends distributed by firms. The households also take into consideration the employment dynamics of the three types of workers. The high educated workers in complex occupations in period \( t + 1 \) are comprised of those who are not exogenously separated in period \( t \) according to the separation rate \( \chi^hc \), in addition to the new matches from the searchers pool (whether they are high educated unemployed or on-the-job searchers):

\[ N^hc_{t+1} = (1 - \chi^hc) N^hc_t + P^hc_t (S^hc_t U^h_t + O_t N^hs_t). \] (5)

Similarly, the high educated workers in simple occupations in period \( t + 1 \) are comprised of those who are neither separated from simple occupations exogenously in period \( t \) according to the separation rate \( \chi^hs \) nor matched with complex occupations as a result of on-the-job search, in addition to new matches from the searchers pool of the high educated unemployed:

\[ N^hs_{t+1} = (1 - \chi^hs) (1 - O_t P^hc_t) N^hs_t + P^hs_t (S^hs_t U^h_t). \] (6)

Finally, the low educated workers in simple occupations in period \( t + 1 \) are comprised of those who are not exogenously separated in period \( t \) according to the separation rate \( \chi^ls \), in addition to the new matches from the searchers pool of the low educated unemployed:

\[ N^ls_{t+1} = (1 - \chi^ls) N^ls_t + P^ls_t (S^ls_t U^l_t). \] (7)

The constant separation rates are justified by Hall (2005), who concludes that over the past 50 years, job separation rates remained almost constant in the United States and by Shimer (2005), who demonstrates that separation rates exhibit acyclicity. \( P^hc_t = \frac{M^hc}{S^hc_t U^h_t + O_t N^hs_t} \) is the probability that a high educated searcher is matched with a complex occupation. \( P^hs_t = \frac{M^hs}{S^hs_t U^h_t} \) is the probability that a high educated searcher is matched with a simple occupation. \( P^ls_t = \frac{M^ls}{S^ls_t U^l_t} \) is the probability that a low educated searcher is matched with a simple occupation. \( M^hc_t = M^hc (V^c_t, S^hc_t U^h_t + O_t N^hs_t) \), \( M^hs_t = M^hs (Z_t, V^s_t, S^hs_t U^h_t) \), and \( M^ls_t = M^ls_t (1 - Z_t, V^c_t, S^ls_t U^l_t) \) represent the number of complex matches, the number of simple matches with the high educated, and the number of simple matches with the low educated, respectively. They are constant returns to scale, homogeneous of degree one functions of the number of corresponding vacancies, \( V^c_t \) and \( V^s_t \), and effective searchers. \( Z_t \) is the proportion of simple vacancies directed to the high educated. The representative household chooses consumption, such that the marginal utility of consumption equals the Lagrange multiplier \( \lambda_c \).
The household chooses the optimal proportion of time the high educated unemployed allot to search for a complex occupation $S_{th}^{hc}$, such that the disutility from increasing search by one unit is offset by the discounted expected value of an additional high educated in a complex occupation:

$$\frac{\partial \Omega_t}{\partial S_{th}^{hc}} + \beta p_{th}^h E_t \left[ \frac{\partial \Gamma_{th}^{hc}}{\partial N_{th}^{hc}} \right] = 0. \hspace{1cm} (9)$$

The household chooses the optimal proportion of time the high educated unemployed allot to search for a simple occupation $S_{th}^{hs}$, such that the disutility from increasing search by one unit is offset by the discounted expected value of an additional high educated in a simple occupation:

$$\frac{\partial \Omega_t}{\partial S_{th}^{hs}} + \beta p_{th}^h E_t \left[ \frac{\partial \Gamma_{th}^{hs}}{\partial N_{th}^{hs}} \right] = 0. \hspace{1cm} (10)$$

The household chooses the optimal proportion of time the low educated unemployed allot to search for a simple occupation $S_{tl}^{hs}$, such that the disutility from increasing search by one unit is offset by the discounted expected value of an additional low educated in a simple occupation:

$$\frac{\partial \Omega_t}{\partial S_{tl}^{hs}} + \beta p_{th}^l E_t \left[ \frac{\partial \Gamma_{tl}^{hs}}{\partial N_{tl}^{hs}} \right] = 0. \hspace{1cm} (11)$$

The household chooses on-the-job search intensity $O_t$, such that the disutility from increasing search by one unit is offset by the difference between the discounted expected value to the household from an additional high educated worker in a complex occupation and that of an additional high educated worker in a simple occupation:

$$\frac{\partial \Omega_t}{\partial O_t} + p_{th}^h \beta E_t \left[ \frac{\partial \Gamma_{th}^{hc}}{\partial N_{th}^{hc}} \right] - p_{th}^l \beta E_t \left[ \frac{\partial \Gamma_{th}^{hs}}{\partial N_{th}^{hs}} \right] (1 - \chi_{th}^{hs}) = 0. \hspace{1cm} (12)$$

From the envelope theorem, an additional high educated matched with a complex occupation accrue a value to the household that is given by

$$\frac{\partial \Gamma_{th}^{hc}}{\partial N_{th}^{hc}} = \Omega^{hc} (1 - H^{hc}) - \Omega^h (1 - S_{th}^{hc} - S_{th}^{hs}) + \lambda_t W_{th}^{hc} H_{th}^{hc}$$

$$+ \beta (1 - \chi^h - p^c S_{th}^{hc}) E_t \left[ \frac{\partial \Gamma_{th}^{hc}}{\partial N_{th}^{hc}} \right] - \beta p_{th}^h S_{th}^{hs} E_t \left[ \frac{\partial \Gamma_{th}^{hs}}{\partial N_{th}^{hs}} \right]. \hspace{1cm} (13)$$
Similarly, an additional high educated matched with a simple occupation accrue a value to the household that is given by

$$\frac{\partial \Gamma^H_t}{\partial N_{ht}} = \Omega^h_h (1 - H^h_t - O_t) - \Omega^h (1 - S^h - S^h_t) + \lambda_t W^h_t H^h_t$$

$$+ \beta (1 - \chi^h (1 - O_t P^h_{t+1} - P^h_t S^h_t) E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{ht}} \right]$$

$$+ \beta (P^h_t O_t - P^h_t S^h_t) E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{ht}} \right].$$

Finally, an additional low educated matched with a simple occupation accrue a value to the household that is given by

$$\frac{\partial \Gamma^H_t}{\partial N_{lt}} = \Omega^l_h (1 - H^l_t) - \Omega^l (1 - S^l_t) + \lambda_t W^l_t H^l_t + (1 - \chi^l - P^l_t S^l_t) E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{lt}} \right].$$

Substituting the envelope conditions into the first-order conditions yields the following representative household’s optimal conditions:

$$\frac{\tau^h}{\beta P^h_{lt}} = -\tau^h E_t (1 - S^h_{t+1} - S^h_{t+1}) + E_t [\Omega^h_h (1 - H^h_{t+1})] + E_t [\frac{H^h_{t+1} W^h_{t+1}}{C_{t+1}}]$$

$$+ E_t \left[ (1 - \chi^h - P^h_{t+1} S^h_{t+1}) \left( \frac{\tau^h}{P^h_{lt}} \right) \right] - E_t [\tau^h S^h_{t+1}],$$

$$\frac{\tau^l}{\beta P^l_{lt}} = -\tau^l E_t (1 - S^l_{t+1} - S^l_{t+1}) + E_t [\Omega^l_h (1 - H^l_{t+1} - O_{t+1})] + E_t [\frac{H^l_{t+1} W^l_{t+1}}{C_{t+1}}]$$

$$+ E_t \left[ \tau^l (O_{t+1} - S^l_{t+1}) \right] + E_t \left[ (1 - \chi^l - P^l_{t+1} S^l_{t+1}) (P^l_{t+1} - S^l_{t+1}) \left( \frac{\tau^l}{P^l_{lt}} \right) \right],$$

$$\frac{\tau^l}{\beta P^l_{lt}} = -\tau^l E_t (1 - S^l_{t+1} - S^l_{t+1}) + E_t [\Omega^l_h (1 - H^l_{t+1})]$$

$$+ E_t \left[ \frac{H^l_{t+1} W^l_{t+1}}{C_{t+1}} \right] + E_t \left[ (1 - \chi^l - P^l_{t+1} S^l_{t+1}) \left( \frac{\tau^l}{P^l_{lt}} \right) \right],$$

where \( \tau^h \) and \( \tau^l \) are the marginal utilities of leisure of the high and low educated unemployed, respectively.

**Firms**

The representative firm chooses the number of complex and simple vacancies to post, in addition to the proportion of the simple vacancies directed to the high educated, in order to
maximize the discounted expected infinite sum of its future profit streams. The profit function
is given by the difference between the value of its production, where the price of one unit of
output is normalized to one, and the total cost incurred for creating the two types of vacancies,
as well as the wages of the three labor types. Assuming the firm has the following value function, \( \Gamma^F_t = \Gamma^F \left( H_t^{hc} N_t^{hc}, H_t^{hs} N_t^{hs}, H_t^{ls} N_t^{ls} \right) \), the optimization problem can be written in the following recursive form:

\[
\begin{align*}
\Gamma^F_t &= \max_{\{v_t, v'_t, v''_t\}} \left\{ Y_t - \omega^c v'^c_t - \omega^s v''_t - N_t^{hc} H_t^{hc} W_t^{hc} - N_t^{hs} H_t^{hs} W_t^{hs} \right. \\
&\quad - N_t^{ls} H_t^{ls} W_t^{ls} + \beta E_t \left[ \frac{\lambda_{t+1}^{hc} + \Gamma^F_{t+1}}{\lambda_t} \right] \left\} \right. ,
\end{align*}
\]

(19)

where \( \omega^c \) is the cost of creating a complex vacancy and \( \omega^s \) is the cost of creating a simple vacancy. The discount factor of firms is given such that it effectively evaluates profits in terms of the values attached to them by households, which ultimately own the firms. Thus, the utility-based and time-varying discount factor used by firms is given by \( (\beta \frac{\lambda_{t+1}^{hc}}{\lambda_t}) \). The maximization is subject to the production function, which is a composite of the complex occupation output \( H_t^{hc} N_t^{hc} \) and the simple occupation output \( H_t^{hs} N_t^{hs} + H_t^{ls} N_t^{ls} \),

\[
Y_t = Y \left[ A_t, \left( H_t^{hc} N_t^{hc}\right), \left( H_t^{hs} N_t^{hs} + H_t^{ls} N_t^{ls} \right) \right],
\]

(20)

where \( A_t \) is aggregate technology. The maximization problem of the firm is also subject to the following employment dynamics:

\[
N_{t+1}^{hc} = (1 - \chi^{hc}) N_t^{hc} + q_t^{hc} V_t^c,
\]

(21)

\[
N_{t+1}^{hs} = (1 - \chi^{hs}) (1 - O_t P_t^{hc}) N_t^{hs} + q_t^{hs} Z_t V_t^s,
\]

(22)

\[
N_{t+1}^{ls} = (1 - \chi^{ls}) N_t^{ls} + q_t^{ls} (1 - Z_t) V_t^s,
\]

(23)

where \( q_t^{hc} = \frac{M_t^{hc}}{V_t^c} \) is the probability of filling a complex vacancy, \( q_t^{hs} = \frac{M_t^{hc}}{V_t^s} \) is the probability that a simple vacancy is filled by a high educated, and \( q_t^{ls} = \frac{M_t^{hc}}{(1 - Z_t) V_t^s} \) is the probability that a simple vacancy is filled by a low educated. The firm chooses the optimal level of complex vacancies to post \( V_t^c \) such that the expected marginal cost of posting this type of vacancy is equal to the discounted expected value for the firm of an additional high educated worker in a complex occupation:

\[
\frac{\omega^c}{q_t^{hc}} = \beta E_t \left[ \frac{\lambda_{t+1}^{hc} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_t^{hc}} \right].
\]

(24)

The firm chooses the optimal level of simple vacancies to post \( V_t^s \) such that the cost of posting a simple vacancy is equal to the discounted expected value of creating an occupation from this vacancy, whether it is filled by a high or a low educated worker:

\[
\omega^s = q_t^{hs} Z_t \beta E_t \left[ \frac{\lambda_{t+1}^{hs} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_t^{hs}} \right] + q_t^{ls} (1 - Z_t) \beta E_t \left[ \frac{\lambda_{t+1}^{ls} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_t^{ls}} \right].
\]

(25)
The firm chooses the optimal proportion of simple vacancies directed to the high educated $Z_t$ such that the discounted expected value of an additional high educated worker in a simple occupation is equal to the discounted expected value of an additional low educated worker in a simple occupation:

$$q_t^{hs}E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_t^{F}}{\partial N_t^{hs}} \right] = q_t^{ls}E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_t^{F}}{\partial N_t^{ls}} \right].$$  \hfill (26)

From the envelope theorem, the value of an additional high educated worker in a complex occupation for the firm is given by the difference between its marginal productivity and the wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated:

$$\frac{\partial \Gamma_t^{F}}{\partial N_t^{hc}} = \frac{\partial Y_t}{\partial N_t^{hc}} - H_t^{hc} W_t^{hc} + (1 - \chi^{hc}) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_t^{F}}{\partial N_t^{hc}} \right].$$  \hfill (27)

Similarly, the value of an additional high educated worker in a simple occupation for the firm is given by the difference between its marginal productivity and the wage, in addition to the discounted expected value of the match in case the worker is neither exogenously separated nor matched with a complex occupation as a result of on-the-job search:

$$\frac{\partial \Gamma_t^{F}}{\partial N_t^{hs}} = \frac{\partial Y_t}{\partial N_t^{hs}} - H_t^{hs} W_t^{hs} + (1 - \chi^{hs}) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_t^{F}}{\partial N_t^{hs}} \right].$$  \hfill (28)

Finally, the value of an additional low educated worker in a simple occupation for the firm is given by the difference between its marginal productivity and the wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated:

$$\frac{\partial \Gamma_t^{F}}{\partial N_t^{ls}} = \frac{\partial Y_t}{\partial N_t^{ls}} - H_t^{ls} W_t^{ls} + (1 - \chi^{ls}) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_t^{F}}{\partial N_t^{ls}} \right].$$  \hfill (29)

Substituting the envelope conditions into the first-order conditions yields the firm’s optimal conditions

$$\frac{\omega^f}{q_t^{hc}} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_t}{\partial N_t^{hc}} - H_t^{hc} W_t^{hc} + (1 - \chi^{hc}) \frac{\omega^c}{q_t^{hc+1}} \right) \right],$$  \hfill (30)

$$\frac{\omega^f}{q_t^{hs}} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_t}{\partial N_t^{hs}} - H_t^{hs} W_t^{hs} + (1 - \chi^{hs}) (1 - O_t^{Pc^h}) \frac{\omega^f}{q_t^{hs+1}} \right) \right],$$  \hfill (31)

$$\frac{\omega^f}{q_t^{ls}} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_t}{\partial N_t^{ls}} - H_t^{ls} W_t^{ls} + (1 - \chi^{ls}) \frac{\omega^f}{q_t^{ls+1}} \right) \right].$$  \hfill (32)
Wages and Hours

In equilibrium, matched firms and workers obtain from the match a total return that is strictly higher than the expected return of unmatched firms and workers because if they separate each will have to go through an expensive and time-consuming process of search before being matched again. We assume that a realized match shares this surplus. Therefore, the wage of a high educated worker in a complex occupation is given by \(^2\)

\[
H_t^{hc} W_t^{hc} = (1 - \xi^{hc}) \left( \frac{\partial Y_t}{\partial N_t^{hc}} + P_t^{hc} S_t^{hc} \frac{\partial \ell_t}{\partial q_t^{hc}} \right) + \zeta^{hc} C_t \left[ \Omega^{h} (1 - S_t^{hc} - S_t^{hs}) - \Omega^{hs} (1 - H_t^{hc} - O_t) - (O_t - S_t^{hc}) \right],
\]

where \(\zeta^{hc}\) is the firm’s share of the surplus. The wage is a weighted average of two terms: The first indicates that the worker is rewarded by a fraction \((1 - \xi^{hc})\) of both the firm’s revenues from the worker’s productivity and the discounted expected value of the match. The second term indicates that the worker is compensated by a fraction \(\zeta^{hc}\) for the forgone benefit from the worker’s outside option or the difference between the leisure of a high educated unemployed and that of a high educated in a complex occupation, in addition to the forgone benefit from being matched with a simple vacancy. Similarly, the wage of the high educated in a simple occupation is given by \(^3\)

\[
H_t^{hs} W_t^{hs} = (1 - \xi^{hs}) \left( \frac{\partial Y_t}{\partial N_t^{hs}} + P_t^{hs} S_t^{hs} \frac{\partial \ell_t}{\partial q_t^{hs}} \right) + \zeta^{hs} C_t \left[ \Omega^{h} (1 - S_t^{hc} - S_t^{hs}) - \Omega^{hs} (1 - H_t^{hc} - O_t) - (O_t - S_t^{hc}) \right],
\]

where \(\zeta^{hs}\) is the firm’s share of the surplus. The wage is a weighted average of two terms: The first indicates that the worker is rewarded by a fraction \((1 - \xi^{hs})\) of both the firm’s revenues from the worker’s productivity and the discounted expected value of the match for the firm. The second term indicates that the worker is compensated by a fraction \(\zeta^{hs}\) for the outside options or the difference between the leisure of a high educated unemployed and that of a high educated in a simple occupation, in addition to the forgone benefit from being matched with a complex vacancy. Finally, the bargained wage of the low educated in a simple occupation is given by \(^4\)

\[
H_t^{ls} W_t^{ls} = (1 - \xi^{ls}) \left( \frac{\partial Y_t}{\partial N_t^{ls}} + P_t^{ls} S_t^{ls} \frac{\partial \ell_t}{\partial q_t^{ls}} \right) + \zeta^{ls} C_t \left[ \Omega^{h} (1 - S_t^{hc} - S_t^{hs}) - \Omega^{hs} (1 - H_t^{hc} - O_t) \right],
\]

where \(\zeta^{ls}\) is the firm’s share of the surplus. The wage is a weighted average of two terms: The first indicates that the worker is rewarded by a fraction \((1 - \xi^{ls})\) for the firm’s revenues from the worker’s productivity and the discounted expected value of the match for the firm. The second term indicates that the worker is compensated by a fraction \(\zeta^{ls}\) for the outside options or the difference between the leisure of a low educated unemployed and that of a low educated in a simple occupation. Finally, the skill premium is defined as the ratio of the weighted average wage of the two high educated types of workers, \(W_t^h = \frac{N_t^{hc} H_t^{hc} W_t^{hc} + N_t^{hs} H_t^{hs} W_t^{hs}}{N_t^{hc} H_t^{hc} + N_t^{hs} H_t^{hs}}\), to the wage of the

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\(^2\) Detailed derivations are included in Appendix 2.

\(^3\) Detailed derivations are included in Appendix 2.

\(^4\) Detailed derivations are included in Appendix 2.
low educated in simple occupations:

\[ \text{Premium}_t = \frac{W^h_t}{W^l_t} \]  

(36)

The hours of the high educated in complex occupations are chosen such that the disutility of leisure from increasing the hours of work by one unit is offset by the increase in marginal productivity due to an increase in hours by one unit:\(^5\)

\[ \frac{\partial (\frac{\partial Y_t}{\partial N^h_t})}{\partial H^c_t} + \frac{\partial Y^c_t}{\partial H^c_t} = 0. \]  

(37)

The hours of the high educated in simple occupations are chosen such that the disutility of leisure from increasing the hours of work by one unit is offset by the increase in marginal productivity due to an increase in hours by one unit:\(^6\)

\[ \frac{\partial (\frac{\partial Y_t}{\partial N^h_t})}{\partial H^s_t} + \frac{\partial Y^s_t}{\partial H^s_t} = 0. \]  

(38)

The hours of the low educated in simple occupations are chosen such that the disutility of leisure from increasing the hours of work by one unit is offset by the increase in marginal productivity due to an increase in hours by one unit:\(^7\)

\[ \frac{\partial (\frac{\partial Y_t}{\partial N^l_t})}{\partial H^l_t} + \frac{\partial Y^l_t}{\partial H^l_t} = 0. \]  

(39)

Finally, the crowding-out effect is defined as

\[ \text{Crowding}_t = \frac{N^h_t H^h_t}{N^h_t H^h_t + N^l_t H^l_t}. \]  

(40)

To close the model, we have

\[ Y_t = C_t + \omega^s V^c_t + \omega^l V^l_t. \]  

(41)

4. Calibration

The functional forms are determined, and the parameters are calibrated in order to solve the model numerically. The instantaneous utility function of consumption is represented by the

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\(^5\) Detailed derivations are included in Appendix 2.

\(^6\) Detailed derivations are included in Appendix 2.

\(^7\) Detailed derivations are included in Appendix 2.
logarithm of consumption expenditures, \( \omega(C_l) = \ln (C_l) \). The instantaneous utility functions of leisure are given by 
\[ \Omega^L = \tau^L(1 - S^h - S^s), \quad \Omega^L = \tau^L(1 - S^i), \quad \Omega^h = \tau^h(1 - H^h), \]
\[ \Omega^s = \tau^s(1 - H^s - O_i), \quad \text{and} \quad \Omega^s = \tau^s(1 - H^s). \]
The matching functions for the complex and simple occupations are represented as a Cobb-Douglas specification with constant returns to scale and are given by 
\[ M^h = M^h(V)^{\gamma}, \quad (S^h U^h)^{1-\gamma}, \quad M^s = M^s(V^s)^{\gamma}, \quad (S^s U^s)^{1-\gamma}, \]
where \( \gamma \in (0, 1) \) is the elasticity of matching with respect to vacancies, respectively, while \( M^h \) and \( M^s \), and \( M^i \) are the level parameters of the matching functions, which capture all factors that influence the efficiency of matching. The technological constraints faced by the firm are also represented by a constant returns to scale Cobb-Douglas function 
\[ Y_t = A_t H^h N^h + H^s N^s + H^s N^s \]
where \( \mu \in (0, 1) \) is the elasticity of output with respect to the complex occupation output. The logarithm of the aggregate technology, \( A_t \), is assumed to follow an AR(1) process as follows:
\[ \log A_{t+1} = \rho^A \log A_t + \varepsilon_{t+1}^A, \quad (42) \]
where \( \varepsilon_{t+1}^A \) is an independently and identically distributed random variable drawn from a normal distribution, with mean zero and standard deviation denoted by \( \sigma_{\varepsilon^A} \).

In this context, numerical values are assigned to the structural parameters in order to conduct a quantitative analysis. Since information may not be available for some parameters, their values are computed from the steady-state values of quantifiable variables. The steady-state values for certain variables are calculated from the averages in the data during the period under study. For instance, the proportions of the employed types are set at \( N^h = 0.15, \) \( N^s = 0.32, \) and \( N^i = 0.2, \) and the premium is 1.5. Table 4 shows the values chosen for the parameters of the model. In this context, some of the parameters are set, as is standard in the literature, and the remainder are solved for.

The first group includes those related to households, such as the proportion of the high educated in the population \( \delta \), which is set at 0.5, close to the data average of the year 2004, given by 0.5067, as well as the household’s discount factor \( \beta \), given by 0.98, which is standard in the literature. The parameters in the utility of leisure for the high educated unemployed \( \tau^i \) is given by 1, for the low educated unemployed \( \tau^L \) is given by 0.7, for the high educated in complex occupations \( \tau^{hc} \) is given by 1.5, for the high educated in simple occupations \( \tau^{hs} \) is given by 0.7, and for the low educated in simple occupations \( \tau^s \) is given by 0.1. The second set pertains to the matching technology, where the level parameters in the matching functions \( M^h, \)
\[ M^s, \] and \( M^i \) are given by 0.3, 0.15, and 0.05, respectively. We follow Krause and Lubik (2004) in setting the elasticity of matches with respect to vacancies \( \gamma \) to 0.6. The separation rates \( \chi^h, \)
\[ \chi^s, \] and \( \chi^i \) from the complex and simple occupations are given by 0.35, 0.02, and 0.02, respectively. These are selected such that their average is close to the weighted average separation rate of 3.23%, calculated by Hall (2005). The third set includes the technological parameters, where the elasticity parameter in the production function \( \mu \) is given by 0.5, as in Krause and Lubik (2004). The autoregressive coefficient in the technological law of motion \( \rho^A \) is given by 0.9, and the standard deviation \( \sigma_{\varepsilon^A} \) is given by 0.0049, consistent with Krause and Lubik (2004). The fourth set includes the parameters in the wage rules, where the firm’s share of the surplus \( \xi^h, \xi^s, \) and \( \xi^i \) are set at 0.59, 0.5, and 0.5, respectively. Finally, the costs of creating the complex vacancy \( \omega^f \) and the simple vacancy \( \omega^s \) are given by 1.043 and 0.08, respectively. The magnitude of difference between the creation costs of the two types of vacancies is consistent with Collard, Fonseca, and Munoz (2003).
5. Analysis

The model is solved by computing the nonstochastic steady state around which the equation system is linearized. The resulting model is solved by the methods developed in Sims (2002). The impulse responses in Figures 1 and 2 show the dynamic evolution of the variables of interest along with a deviation of output from its long-run trend as a consequence of an aggregate technological shock.

The technological shock increases the productivity of all types of workers. This increases the discounted expected value of an additional worker of any type to the firm. The firm posts simple and complex vacancies such that the expected marginal cost of posting each type of vacancy is equal to the discounted expected value for the firm of an additional worker. Accordingly, the increase in the marginal productivity of workers induces firms to increase their posting of both simple and complex vacancies. However, as complex vacancies are more costly to create compared to simple vacancies, the increase in the former lags that of the latter.

According to the impulse responses, the increase in simple vacancies is instantaneous. On the other hand, the proportion of simple vacancies directed to the high educated declines with the cycle. These two factors cause the probability that a low educated will be matched with a simple occupation to increase and thus induce the low educated unemployed to increase their search intensity for simple vacancies. This causes an increase in the employment of the low educated and a decrease in the unemployment of this type of worker.

On the other hand, the high educated unemployed increase their search for a simple occupation slowly as the proportion of these vacancies directed toward them increases. This
causes an increase in the employment of the high educated in simple occupations, especially as
the separation due to on-the-job search declines. This decline in on-the-job search intensity is
due to the lagged increase in the complex vacancies. Even though the high educated
unemployed increase their search intensity for the complex vacancies instantaneously, the
matching of this type is delayed, causing a delay in the increase in their employment. This is
caused because of the lagged increase in the creation of complex vacancies and the lagged
increase in on-the-job search intensity.

As the number of complex vacancies increases, the probability that a high educated is
matched with a complex vacancy increases, and accordingly, on-the-job search intensity
increases. This increase in on-the-job search intensity adversely impacts the hours of work of
the high educated in simple occupations. As the high educated in simple occupations increase
the portion of time spent to search for a complex vacancy, their hours of work decline. This
causes the total hours of the high educated in simple occupations to decline. On the other hand,
because of the increase in the employment and hours of work of the high educated in complex
occupations, the total hours of this type increase. The wage of the high educated is a weighted
average of the wage of the high educated in complex and in simple occupations. The weights are
given by the total hours of each type. Accordingly, these two factors cause the weight of the
high educated in complex occupations to increase with a lag relative to all the high educated
employed. This causes the wage of the high educated to increase with a lag, causing the skill
premium to increase with a lag as well.

Comparing the moments of the model in Table 6 to the data observations, the model
succeeds in several aspects. For instance, the model replicates the lagged procyclicality of the

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*Figure 2. Impulse Response Functions to a 1% Aggregate Technological Shock*
skill premium; however, the lag is shorter in the model than in the data. The model also replicates the procyclicality of the total hours of the low educated in simple occupations and the lagged countercyclicality of the unemployment ratios of the high and the low educated as well. The model, however, does not replicate the lagged increase in the total hours of the high educated in complex occupations, which have driven the lagged procyclicality of the skill premium in the data. In the model, the hours of the high educated in simple occupations decline because of the increase in on-the-job search intensity, and that is the critical factor that drives the lagged procyclicality of the skill premium. The model also produces a countercyclical crowding-out effect.

6. Conclusion

This article attempts to enhance our understanding of the economic fluctuations in a labor market with heterogeneous agents and can be perceived as a contribution to a literature that devoted considerable attention to documenting and analyzing the cyclical behavior of labor market variables that reflect agent heterogeneity across observable skills or educational levels. A set of stylized facts imply that the skill premium is uncorrelated with contemporaneous output and lags the business cycle. In addition, an economic expansion is accompanied contemporaneously by a rise in the total hours of all labor types in simple occupations and followed with a lag by an increase in the total hours of all those employed in complex occupations and a decrease in the crowding out of the low educated by the high educated in occupying simple jobs. These observations might be intuitively interpreted to reflect a lagged cyclical upgrading of jobs by the high educated from a simple to a complex occupation, which causes the gap between their wage and that of the low educated to widen. In order to comprehend the factors behind the evolution of these patterns, a model is developed where workers of heterogeneous education levels search for two types of vacancies that are distinguished by their educational requirements. On-the-job search is allowed. An aggregate technological shock induces firms to increase their posting of simple and complex vacancies. However, as complex vacancies are more costly to create compared to simple ones, the increase in the former lags that of the latter. As the number of complex vacancies increase, on-the-job search intensity increases. As the high educated in simple occupations increase the portion of time spent to search for a complex vacancy, their hours of work decline. On the other hand, because of the lagged increase in the employment and hours of work of the high educated in complex occupations, the total hours of this type increase. These two factors cause the weight of the high educated in complex occupations to increase relative to all the high educated employed. This causes the gap between the wage of the high educated and that of the low educated to increase with a lag, causing the skill premium to increase with a lag as well. The success of this model is attributed to the additional dynamics that it introduces, such as competition between those distinguished by their educational levels for a job with a particular educational requirement, the crowding out of the unsuccessful by the successfully matched, and the possibility of a mismatch between the educational level of the successful and the educational requirement of the job they occupy. This serves as an incentive for the mismatched to continue searching for a job that better suits their qualifications, allowing for job-to-job mobility. This cyclical upgrading of jobs can explain the lagged procyclicality of the skill premium. Possible
Table 1. Extracted Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MONTH</td>
<td>Month of interview</td>
</tr>
<tr>
<td>BEMP2</td>
<td>Employed persons, excluding farm and private household workers</td>
</tr>
<tr>
<td>GRDHI</td>
<td>Highest grade attended</td>
</tr>
<tr>
<td>GRDATN</td>
<td>Educational attainment</td>
</tr>
<tr>
<td>OCC</td>
<td>Occupation of job last week</td>
</tr>
<tr>
<td>ERNWKC</td>
<td>Weekly earnings before deductions (1979–1988)</td>
</tr>
<tr>
<td>HOURS</td>
<td>Total hours worked last week</td>
</tr>
<tr>
<td>ERNWGT</td>
<td>Earnings weight</td>
</tr>
</tbody>
</table>

extensions to the model include the introduction of investment in physical capital. This is expected to have interesting implications, as it provides the households with an alternative channel for intertemporal consumption smoothing other than creating vacancies and thus can have an effect on the creation and composition of vacancies in the economy.

Appendix 1: Data

The data set used is the Outgoing Rotation Group of the Current Population Survey. The Current Population Survey is a rotating panel. After the fourth month in the survey, the participants take an eight-month hiatus. Afterward, they are interviewed for another four months, and after the eighth month in the sample, they are completely dropped from the survey. The Outgoing Rotation series is a merged collection of the fourth and eighth month-in-sample groups from all 12 months. These two groups play a special role, as they are given additional questions, the answers to which are collected in the Outgoing Rotation Group files. The data are monthly and cover the period from January 1979 until December 2004. At the end of each year, the 12 monthly files from January until December are concatenated into a single annual file. The variables extracted are as shown in Table 1.

Each annual file is divided into monthly files according to the variable MONTH. For each monthly file, participants are split into those employed and those unemployed according to BEMP2. This distinction takes into consideration participation in the labor force. Both the employed and the unemployed are further split into high educated and low educated households, where the high educated are the workers who obtained some college education or higher. Table 2 shows the variables’ ranges defining the high and low educated.

Each worker group, the high or the low educated, is further divided into two groups: those employed in complex occupations and those employed in simple occupations, where the former are jobs that require at least some college education. This mapping between occupations and educational requirements is based on judgment. In most cases, it is straightforward to determine whether an occupation requires college education. In the cases where it is not clear, the occupations are considered once as complex and once as simple. The results did not change in both cases. The complex and simple occupations are defined by the ranges of the variable OCC, specified as shown in Table 3.

Therefore, we have four employed and two unemployed types: the high educated employed in a complex occupation, the high educated employed in a simple occupation, the high educated unemployed, the low educated employed in a complex occupation, the low educated employed in a simple occupation, and the low educated unemployed.

The weighted average of the weekly earnings and hours worked last week for each of the working groups are calculated using the proper weights ERNWGT. These weights are created for each month such that, when applied, the resulting counts are representative of the national counts. Thus, the proper application of weights enables the results to

Table 2. Ranges for the High and Low Education Levels

<table>
<thead>
<tr>
<th>Period</th>
<th>High Educated</th>
<th>Low Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979–1988</td>
<td>14 ≤ GRDHI ≤ 19</td>
<td>1 ≤ GRDHI ≤ 13</td>
</tr>
<tr>
<td>1989–1991</td>
<td>13 ≤ GRDHI ≤ 18</td>
<td>1 ≤ GRDHI ≤ 12</td>
</tr>
<tr>
<td>1992–2004</td>
<td>40 ≤ GRDATN ≤ 46</td>
<td>31 ≤ GRDATN ≤ 39</td>
</tr>
</tbody>
</table>
Table 3. Ranges for the Complex and Simple Occupation Types

<table>
<thead>
<tr>
<th>Period</th>
<th>Complex Occupation</th>
<th>Simple Occupation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979–1982</td>
<td>1–85</td>
<td>86–90</td>
</tr>
<tr>
<td></td>
<td>91–96</td>
<td>100–101</td>
</tr>
<tr>
<td></td>
<td>102–246</td>
<td>260–995</td>
</tr>
<tr>
<td></td>
<td>178–242</td>
<td>243–991</td>
</tr>
<tr>
<td>1992–2002</td>
<td>0–163</td>
<td>164–165</td>
</tr>
<tr>
<td></td>
<td>166–173</td>
<td>174–177</td>
</tr>
<tr>
<td></td>
<td>178–242</td>
<td>243–999</td>
</tr>
<tr>
<td></td>
<td>2100–3650</td>
<td>3700–9830</td>
</tr>
</tbody>
</table>

be presented in terms of the population of the United States as a whole instead of just the participants in the survey. The hourly wage of each worker type is calculated as the ratio of the weighted average weekly earnings to the weighted average hours worked last week for each group. These derived wages are used to calculate the skill premium, which is defined as the ratio of the weighted average hourly wage of the two types that are high educated to that of the low educated in simple occupations. To calculate measures of employment and unemployment ratios, the binary variable

Table 4. Calibration of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.5</td>
<td>Proportion of the high educated in the population</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>Household discount factor</td>
</tr>
<tr>
<td>$\chi_{hc}$</td>
<td>0.35</td>
<td>Separation rate from complex occupations</td>
</tr>
<tr>
<td>$\chi_{hs}$</td>
<td>0.02</td>
<td>Separation rate of high educated from simple occupations</td>
</tr>
<tr>
<td>$\chi_{ls}$</td>
<td>0.02</td>
<td>Separation rate of low educated from simple occupations</td>
</tr>
<tr>
<td>$M_{hc}$</td>
<td>0.3</td>
<td>Efficiency in the complex occupation matching function</td>
</tr>
<tr>
<td>$M_{hs}$</td>
<td>0.15</td>
<td>Efficiency in the simple occupation matching function with the high educated</td>
</tr>
<tr>
<td>$M_{ls}$</td>
<td>0.05</td>
<td>Efficiency in the simple occupation matching function with the low educated</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.6</td>
<td>Elasticity of complex matches with respect to complex vacancies</td>
</tr>
<tr>
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Table 5. Compiled Data

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*TH^{bc} = total hours of the high educated in complex occupations; TH^{bs} = total hours of the high educated in simple occupations; TH^s = total hours of the low educated in simple occupations; p-values in parentheses.*
Table 6. Model Moments

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$TH_{bc}$ = total hours of the high educated in complex occupations; $TH_{hs}$ = total hours of the high educated in simple occupations; $TH_{ls}$ = total hours of the low educated in simple occupations; $p$-values in parentheses.
BEMP2 is used to distinguish the two groups. The unemployed are divided into high and low educated in the same manner as explained earlier. The employed are divided into four types as explained earlier. The ratios of the employed and the unemployed types to the total sample are calculated by summing over the weights in each type and dividing by the sum of the weights of the total sample. The total hours are calculated by multiplying the level of employment in every type by the weighted average weekly hours of work for each type. A crowding-out variable is calculated as the proportion of the total hours of the high educated among the total hours of all those employed in simple occupations. Finally, the variables compiled and used in the analysis are (i) the skill premium, (ii) the proportion and the hours of the high educated in complex occupations, (iii) the total hours of the high educated employed in complex occupations, (iv) the proportion and the hours of the high educated in simple occupations, (v) the total hours of the high educated employed in simple occupations, (vi) the proportion and the hours of the low educated in simple occupations, (vii) the total hours of the low educated employed in simple occupations, (viii) the proportion of the high educated unemployed, (ix) the proportion of the low educated unemployed, and (x) crowding out. Finally, the real GDP data (chained dollars, seasonally adjusted at annual rates) is extracted from the National Income and Product Accounts NIPA. As the GDP data are quarterly, these monthly time series are transformed into quarterly ones by taking three-month averages. All variables except employment and unemployment ratios are lagged. The data are seasonally adjusted or deseasonalized using a ratio to moving average multiplicative seasonal filter. All variables are detrended using the Hodrick Prescott filter with a smoothing parameter of 1600.

Appendix 2: Derivations

The Wage of High Educated Workers in Complex Occupations

The wage of the high educated in a complex occupation is determined by

\[ W_{hc} = \text{argmax} \left[ 1 - \frac{\partial F^H}{\partial N_{hc}} \right]^{\frac{1}{\lambda_t}} \left[ \frac{\partial F^H}{\partial N_{hc}} \right]^{\lambda_t}. \]

Then the sharing rule implies \( \psi_{hc} \left[ \frac{\partial F^H}{\partial N_{hc}} \right] = (1 - \psi_{hc}) \lambda_t \left[ \frac{\partial F^H}{\partial N_{hc}} \right]. \) Substituting the envelope conditions of the household \( \frac{\partial F^H}{\partial N_{hc}} \) and of the firm \( \frac{\partial F^F}{\partial N_{hc}}, \) in addition to

\[ \psi_{hc} \left[ \frac{\partial F^H}{\partial N_{hc}} \right] = (1 - \psi_{hc}) \beta \left[ \frac{\partial F^F}{\partial N_{hc}} \right], \]

from the first-order condition yields

\[ \psi_{hc} \left[ -\Omega^H + \Omega^F + \lambda_t W_{hc} H_{hc} + (1 - \chi_{hc} - P^h_{hc} \psi_{hc}) \lambda_t - (1 - \psi_{hc}) \frac{\omega^F}{\theta^F} - \beta P^h_{hc} \psi_{hc} \right] \]

\[ = (1 - \psi_{hc}) \lambda_t \left[ \frac{\partial Y}{\partial N_{hc}} \right] - H_{hc} W_{hc} + (1 - \chi_{hc}) \frac{\omega^F}{\theta^F}. \]

Solving for the equilibrium wage rule for the high educated workers in complex occupations yields Equation 33.

The Wage of High Educated Workers in Simple Occupations

The wage of the high educated in a simple occupation is determined by

\[ W_{hs} = \text{argmax} \left[ 1 - \frac{\partial F^H}{\partial N_{hs}} \right]^{\frac{1}{\lambda_t}} \left[ \frac{\partial F^H}{\partial N_{hs}} \right]^{\lambda_t}. \]

Then the sharing rule implies \( \psi_{hs} \left[ \frac{\partial F^H}{\partial N_{hs}} \right] = (1 - \psi_{hs}) \lambda_t \left[ \frac{\partial F^H}{\partial N_{hs}} \right]. \) Substituting the envelope conditions of the household \( \frac{\partial F^H}{\partial N_{hc}} \) and of the firm \( \frac{\partial F^F}{\partial N_{hc}}, \) in addition to

\[ \psi_{hs} \left[ \frac{\partial F^H}{\partial N_{hs}} \right] = (1 - \psi_{hs}) \beta \left[ \frac{\partial F^F}{\partial N_{hc}} \right], \]

from the first-order condition yields:
\[
\begin{align*}
\text{Equation 34:} & \\
& = (1 - \xi^h)\lambda_t \left[ \frac{\partial Y_t}{\partial N_t^h} - H_t^h W_t^h \right] + \left( 1 - \chi^h \right) \left( 1 - O_t^h \right) \alpha^t \frac{\partial^t}{\partial t^t}.
\end{align*}
\]

Solving for the equilibrium wage rule for the high educated workers in simple occupations yields Equation 34.

The Wage of Low Educated Workers in Simple Occupations

The wage of the low educated in a simple occupation is determined by

\[
W_t^l = \arg\max \left[ 1 + \frac{\partial T^l}{\partial N_t^l} \right] \left( \frac{\partial T^l}{\partial N_t^l} \right) \zeta^l.
\]

Then the sharing rule implies \( \zeta^l \frac{\partial T^l}{\partial N_t^l} = (1 - \xi^l)\lambda_t \frac{\partial T^l}{\partial N_t^l} \). Substituting the envelope conditions of the household and of the firm, in addition to \( \frac{\partial T^l}{\partial N_t^l} \), from the first-order condition yields

\[
\xi^l \left[ -\Omega^l(1 - S^h_t) + \Omega^l(1 - H^h_t) + \lambda_t W_t^h + (1 - \chi^h) \left( 1 - O_t^h \right) \alpha^t \right] = (1 - \xi^l)\lambda_t \left[ \frac{\partial Y_t}{\partial N_t^l} - H_t^l W_t^l \right] + \left( 1 - \chi^l \right) \left( 1 - O_t^l \right) \alpha^t \frac{\partial^t}{\partial t^t}.
\]

Solving for the equilibrium wage for low educated workers in simple occupations yields Equation 35.

The Hours of High Educated Workers in Complex Occupations

The hours of work of the high educated workers in complex occupations are given by

\[
H_t^c = \arg\max \left[ 1 + \frac{\partial T^c}{\partial N_t^c} \right] \left( \frac{\partial T^c}{\partial N_t^c} \right) \zeta^c.
\]

Substituting the envelope conditions for \( \frac{\partial T^c}{\partial N_t^c} \) and \( \frac{\partial T^c}{\partial N_t^c} \), the hours are thus given by Equation 37.

The Hours of High Educated Workers in Simple Occupations

The hours of work of the high educated workers in simple occupations are given by

\[
H_t^s = \arg\max \left[ 1 + \frac{\partial T^s}{\partial N_t^s} \right] \left( \frac{\partial T^s}{\partial N_t^s} \right) \zeta^s.
\]

Substituting the envelope conditions for \( \frac{\partial T^s}{\partial N_t^s} \) and \( \frac{\partial T^s}{\partial N_t^s} \), the hours are thus given by Equation 38.

The Hours of Low Educated Workers in Simple Occupations

The hours of work of the low educated workers in simple occupations are given by

\[
H_t^l = \arg\max \left[ 1 + \frac{\partial T^l}{\partial N_t^l} \right] \left( \frac{\partial T^l}{\partial N_t^l} \right) \zeta^l.
\]

Substituting the envelope conditions for \( \frac{\partial T^l}{\partial N_t^l} \) and \( \frac{\partial T^l}{\partial N_t^l} \), the hours are thus given by Equation 39.
References

Keane, Michael, and Eswar Prasad. 1993. Skill levels and the cyclical variability of employment, hours, and wages. International Monetary Fund Staff Papers 4, 4:711–43.