Problem Set 3

1. One possible mathematical form for a fixed proportions production is

\[ Q = F_n(K,L) = \min(aK,bL) \]

where the term "min" means that Q takes on the lower value of the two terms inside the parentheses. If, for example, aK is less than bL then Q = aK and K is the binding constraint in production.

a. When Q = \min(aK,bL), what is the optimal capital-labor ratio?

b. What are the returns to scale for the above fixed proportions production function?

c. Does the fixed proportions production function \( Q = (\min(aK,bL))^{\frac{1}{2}} \) exhibit constant returns, increasing returns or decreasing returns to scale?

2. Suppose you have a short run production function estimated to be:

\[ Q = 100L + 50L^2 - 3L^3 \]

where L = number of workers. At what number of workers will you first experience diminishing returns? At what level of employment will total output be maximized with this plant size?

3. A contractor for the U.S. Defense Department produces airplanes from capital (K) and labor (L) by the production function:

\[ A = K \times L^2 \]

The marginal product of labor (\( MP_L \)) = 2KL.
The marginal product of capital (\( MP_K \)) = L^2.
The Defense Department is ordering 125 airplanes.

a. How much labor and capital will the firm employ if labor costs twice that of capital? (Hint: Let \( w = \$2 \) and \( r = \$1 \). Use \( MP_L/MP_K = w/r \) to find L in terms of K. Then substitute this into the production function for \( A = 125 \).)

b. How much capital and labor will the firm use if capital costs twice that of labor?
4. Cabela’s, Inc. -- a sporting goods company in Sydney, Nebraska -- is contemplating entering the Boron fly rod industry. There are three types, and only three types, of Boron fly rod manufacturing plants. Each type has the following total cost relationships:

- **Plant 1:** \( TC = 2Q^3 - 600Q^2 + 45150Q \)
- **Plant 2:** \( TC = Q^3 - 400Q^2 + 40200Q \)
- **Plant 3:** \( TC = Q^3 - 200Q^2 + 10200Q \)

   a. Give the equations for each type of plant’s average cost curves and graph those curves. (Hint: To help you in your graphing, find the minimum point of each plant’s average cost curve. This occurs at the quantity (Q) where the first derivative of the average cost equation equals zero. To find where each average cost curve intersects with the other average cost curves, set them equal to each other and solve for Q.)

   b. Graph the long-run average cost curve faced by Cabela’s in the manufacturing of Boron fly rods. How is this curve related to the individual plant’s average cost curves?

   c. You have been hired by Cabela’s to advise them on which type of plant to build. If the executives of Cabela think they will sell the following number of fly rods, which type of plant would you recommend, and why? (Assume that they will build only one plant.)

      (i) 100 fly rods

      (ii) 150 fly rods

      (iii) 200 fly rods

   d. Assume that all firms in the Boron fly rod industry, a competitive industry, adopt the type of plant with the lowest possible average cost. If the demand for Boron fly rods is given by the following equation, how many firms will there be in the industry in the long run? (Let \( P = AC \))

\[
Q = 3000 - 6P
\]

5. Ten firms compete in a market to sell product X. The total sales of all firms selling the product are $2 million. Ranking the firms’ sales from highest to lowest, we find the top four firms’ sales to be $260,000, $220,000, $150,000 and $130,000, respectively. Calculate the four-firm concentration ratio in this market for product X.
6. Suppose the own price elasticity of market demand for retail gasoline is -0.8, the Rothschild Index is 0.5, and a typical gasoline retailer enjoys sales of $1.5 million annually. What is the price elasticity of demand for a representative gasoline retailer’s product?

7. A firm has $1.5 million in sales, a Lerner index of 0.57, and a marginal cost of $50, and competes against 800 other firms in its relevant market.
   a. What price does this firm charge its customers?
   b. By what factor does this firm mark up its price over marginal cost?
   c. Do you think this firm enjoys much market power? Explain.

8. Based on the information given, indicate whether the following industry is best characterized by the model of perfect competition, monopoly, monopolistic competition, or oligopoly.
   a. Industry A has a four-firm concentration ratio of 0.005 percent and a Herfindahl-Hirschman index of 75. A representative firm has a Lerner index of 0.45 and Rothschild index of 0.34.
   b. Industry B has a four-firm concentration ratio of 0.0001 percent and a Herfindahl-Hirschman index of 55. A representative firm has a Lerner index of 0.0034 and Rothschild index of 0.00023.
   c. Industry C has a four-firm concentration ratio of 100 percent and a Herfindahl-Hirschman index of 10,000. A representative firm has a Lerner index of 0.4 and Rothschild index of 1.0.
   d. Industry D has a four-firm concentration ratio of 100 percent and a Herfindahl-Hirschman index of 5,573. A representative firm has a Lerner index of 0.43 and Rothschild index of 0.76.

9. Firms like Papa John’s, Domino’s, and Pizza Hut sell pizza and other products that are differentiated in nature. While numerous pizza chains exist in most locations, the differentiated nature of these firms’ products permits them to charge prices above marginal cost. Given these observations, is the pizza industry most likely a monopoly, perfectly competitive, monopolistically competitive or an oligopoly? Use the causal view of structure, conduct, and performance to explain the role of differentiation in the market for pizza. Then apply the feedback critique to the role of differentiation in the industry.

10. Several years ago Pfizer and Warner-Lambert agreed to a $90 billion merger, thus creating one of the world’s largest pharmaceutical companies. Pharmaceutical companies tend to spend a greater percentage of sales on R&D activities than other industries. The government encourages these R&D activities by granting companies patents for drugs approved by the Food and Drug Administration. For instance, Pfizer-Warner-Lambert spent large sums of money developing its popular cholesterol-lowering drug Lipitor, which is currently protected under a patent. Lipitor sells for about $3 per pill. Calculate the Lerner index if the marginal cost of producing Lipitor is $0.30 per pill. Does the Lerner index make sense in this situation? Explain.