Portfolio Allocation and Asset Returns in an OLG Economy with Increasing Risk Aversion

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Abstract

This paper examines asset returns, equity premium and portfolio allocation in a three-period overlapping generations model with increasing risk aversion (IRRA). This specification allows us to separate the effect of the two risk aversion parameters (middle-aged and old). We find that the effect of IRRA in a life cycle model has the following implications for financial markets: an increase in all security returns, a higher risk premium, a reduced level of saving, and a decline in risky portfolio shares. These results suggest that an economy in which the level of average risk aversion is rising as the population ages would see a drop in the savings rate, a rise in security returns, and a portfolio shift into safer assets. Our findings are driven by fairly small differences in the risk aversion parameters and even small increases in older agents’ risk aversion produces significant results. In addition, we find that the relative difference between the two risk aversions (how much more risk averse old agents are relative to the middle aged) matters more than the average risk aversion in the economy (how much more risk averse both cohorts are).

JEL Classification: G0, G12, D10, E21.

Key Words: Equity premium puzzle, Overlapping generations model, Increasing Risk Aversion, Portfolio allocation.

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1. Introduction

The issue of optimal portfolio allocation over the life cycle has received considerable attention in recent years. This is partly motivated by the steady change in population demographics and market participation rates in advanced economies during the past three decades. Of particular concern is the ageing of the baby boom generation and its impact on asset returns, portfolio allocations, and the level of private and national savings.\(^1\) According to the US Bureau of the Census, the US population over the age of 65, currently at 13\%, is expected to grow to over 20\% by 2030; the proportion of US households headed by someone 65 years or older is expected to increase to roughly 40\% by 2040, compared to 22\% in 1996. In addition, this age cohort is wealthier than the average population: in 2005 the median financial wealth of households aged 65 or older was $40,000 compared to an average median wealth of $15,420 for the other age cohorts.

At the same time, a large body of empirical work has found evidence that risk aversion tends to increase with age. Morin and Suarez (1983) study the effect of age on the specific composition of risky assets in an investor’s portfolio and conclude that risk aversion increases with age. Bakshi and Chen (1994) use asset allocation data post-1945 for the US and document a strong pattern of an increase in risk aversion with age. Using insurance data, Eisenhauer and Halek (1999) report evidence of increasing absolute risk aversion which implies increasing relative risk aversion (IRRA). More recent studies also document a positive relationship between age and relative risk aversion both for industrialized and emerging economies (Palsson (1996), Sung and Hanna (1996), and Eisenhauer and Halek (2001)).\(^2\)

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\(^2\)It should be noted that a few early studies in the literature document either CRRA or DRRA (Friend and Blume (1975), Cohn et al. (1975), Wang and Hanna (1997)). The lack of consensus in the empirical literature appears to be driven partly by the fact that results are highly sensitive to wealth measurements, and partly because age and wealth are highly correlated. Nonetheless, there seems to be a general agreement that risk aversion increases beyond age 65 (retirement age). This is
The growing share of older, wealthier, and more risk averse agents is expected to have significant implications for asset prices, savings, and portfolio allocations. The existing literature in this topic is far from homogeneous, ranging from one extreme view of a potential "market meltdown" associated with the retirement of baby boomers, to another extreme of a very modest – if any – effect of the changing demographics on asset returns and the composition of household balance sheets.3

This paper presents an asset pricing model that investigates the equity premium puzzle and explores agents’ life cycle savings/investment and portfolio allocation decisions. The novelty of this paper comes from introducing increasing risk aversion (IRRA) into the three-period overlapping generations (OLG) model framework of Constantinides, Donaldson, and Mehra (CDM) (2002) with borrowing constraints. There are three age cohorts (young, middle-aged, and old), each facing different sources of uncertainty on wage and equity income; the attractiveness of equity depends on the stage of the life-cycle. The young, for whom equity is a “hedge” against future wage shocks, are constrained from participating in securities markets. In the absence of participation from the young, asset prices are exclusively driven by middle-aged investors for whom equity is less desirable since they do not face wage uncertainty. The borrowing constraint feature of the model increases equity returns (because the middle-aged require a higher premium to hold equity), reduces the risk-free rate (because the young are unable to borrow), and thus increases the equity premium.

Consistent with the empirical evidence, we assume that older agents are more risk averse than middle-aged ones. There are several implications from this set-up. First, the three-period OLG model allows us to separate the effect on security returns, savings, and asset allocation of the two risk aversion parameters. Interestingly, we find that the relative difference between the two risk aversions (how much more risk averse old

3See, for example, Yoo (1994), Poterba (2001), Brooks (2000), Abel (2003), and Geanakoplos, Magill and Quinzii (2004).
agents are relative to the middle aged) matters more than the average risk aversion in the economy (how much more risk averse both cohorts are). It is important to stress that results are driven by fairly small differences in the risk aversion parameters: even a small increase in older agents’ risk aversion produces significant results. Second, the introduction of IRRA leads to a more realistic representation of the economy: our model easily matches the equity premium and explains observed patterns in risky portfolio shares while keeping to a fairly simple framework and without extreme assumptions on background risk. Third, unlike the majority of other works, we study both equity premium and portfolio allocation decisions in a unified framework.

Overall, the effects of IRRA on a life cycle model are significant. Our results suggest that an increase in risk aversion associated with population ageing have the following implications for financial markets: 1) an increase in all security returns with equity returns dominating bond returns, 2) a higher equity premium, 3) reduced level of saving, and 4) reduced risky portfolio shares. It is important to note however that our findings do not support the “market meltdown” hypothesis associated with the retirement of the baby-boomers and the resulting selling pressures on financial markets. The equity premium generated by IRRA is in line with the historical average and the portfolio share of the risky assets is within the 40-50% range. Even in cases when the risk aversion increases substantially more over the life cycle, the portfolio share of risky asset remains around 27%.

Our work is related to two strands of literature. The first one focuses on reconciling the high equity premium observed in the data with theoretical findings of reasonably specified asset pricing models. Studies in this area have proposed several generalizations of the key features of the Mehra and Prescott (1985) model ranging from preference modifications, lower tail risks, survival bias, incomplete markets, market imperfections, limited participation, macroeconomic shocks, and behavioral explanations. This paper falls firmly within this literature and contributes to it by analyzing the equity premium

The second strand of literature deals with household portfolio choices, and comprises both theoretical and empirical studies on the life cycle patterns of asset allocation and participation rates. Early models with complete markets and no labor income predict that the optimal fraction of wealth invested in risky asset is constant, independent of wealth and age, and depends only on the risk aversion and the moments of asset returns (Samuelson (1969) and Merton (1971)). If households are endowed with time invariant risk aversion then the proportion held in risky asset does not vary over the life cycle. Moreover, when calibrated to historical values of asset returns, these models predict that nearly all households should participate in the stock market and that the appropriate portion of wealth placed in risky asset is counter-factually large – sometimes higher than 100%.

However, empirical studies on the life-cycle pattern of household portfolio choice have documented a number of empirical regularities that contradict the predictions of the theory. First, while stock market participation has increased significantly over the decades it is still just slightly above 50% – well below the levels predicted by models (Bertaut and Haliassos (1997), Bertraut and Starr-McCluer (2002)). There is a clear age effect in participation rates with a distinct hump-shaped pattern: participation levels are low at young ages and after retirement (Ameriks and Zeldes (2004), Poterba and Samwick (2001), Faig and Shum (2002), Heaton and Lucas (2000), Gomes and Michaelides (2005)). Second, condition on participation, the share of the risky asset in the portfolio is considerably below 100%: Bertraut and Starr-McCluer (2002) estimate it to be around 54.4%. In addition, a number of recent studies have documented a strong relationship between age and the share invested in the risky asset. For example,
Fagereng (2009) finds that households hold a remarkably stable share of risky assets (around 39%) up until the age of 50, which is then reduced to around 30% by the time of retirement. Andersson (2001) shows that the fraction of risky asset follows a hump-shaped age profile, while the share of the “safe” asset has a distinct U-shaped pattern.\(^5\)

The discrepancy between early theoretical predictions and recent empirical findings has led to the development of a large body of theoretical work which has attempted, with varying degree of success, to reconcile observations with theory. Standard models have been extended to analyze asset allocation decision in both infinite and finite horizon models and include a few key features such as uninsurable labor income risk, preference heterogeneity, market participation costs, precautionary and retirement savings, bequest motives, small probability of disastrous events, and housing investment.\(^6\)

Our model further extends these efforts by introducing IRRA in a life cycle model which delivers equity premium and risky allocation shares that are consistent with the recent empirical evidence without assuming unreasonable parameter values.

The rest of this paper is organized as follows. Section 2 introduces the model. In section 3 we discuss the model calibration. In section 4 we present and discuss our findings. The results from introducing increasing risk aversion are compared to a non-increasing risk aversion setup. Section 5 concludes.

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\(^5\) The empirical relationship between age and portfolio shares of the risky asset is not very clear cut partly because of the identification issues related to age, time, and cohort (birth year) effects. For example, a few studies find that conditional on owning risky any assets, the share of financial wealth held in risky assets is relatively stable with age except for the very young and the very old households (Ameriks and Zeldes (2004), Poterba (2001), Bertraut and Starr-McCluer (2002), and McCarthy (2004)).

2. The Model

We consider the three-period OLG exchange economy of Constantinides, Donaldson, and Mehra (2002), where each generation lives as young, middle-aged, and old. Each consumer-generation is modeled as a representative agent in order to focus on across-generation instead of within-generation heterogeneity. There is only one consumption good which perishes at the end of each period. All prices, wages, consumption, dividends, and coupons payments are quoted in terms of this single consumption good. The model assumes within-generation market completeness (i.e., the existence of a complete set of contingent claims through which agents of the same generation can insure against their income shocks). However, following CDM, we assume two types of market incompleteness: 1) consumers cannot trade claims against their future income with consumers from another generation, and 2) consumers cannot trade with consumers from an unborn generation.

There is a financial market where two types of securities are traded: a bond (riskless asset) and a share of equity (risky asset), both infinitely lived. The bond here works as a proxy for long-term government debt. The bond is default-free and pays a fixed coupon of 1 unit of the consumption good in every period in perpetuity. Its supply is fixed at $b$ units. The aggregate coupon payment is $b$ in every period and represents a portion of the economy’s capital income. $p_t^b$ is the ex coupon price of bond in period $t$.

One perfectly divisible equity share is also traded. The equity is the claim to the net dividend stream $\{d_t\}$, the sum total of all the private capital income (stocks, corporate bonds, and real estate). Similarly, the ex dividend price of equity in period $t$ is $p_t^e$. The total supply of equity is fixed at one.

Following Constantinides, Donaldson, and Mehra (2002), we study the borrowing-constraint version of the model economy in which the young generation is effectively excluded from participating in the financial market. The young born in period $t$ earn deterministic low wages $w^0 > 0$, and want to smooth their lifetime consumption by
borrowing against future wage income, consuming part of the loan, and investing the rest in equity. However, with low first period wages, and with only human capital as collateral, they are unable to borrow against future wage income. The young are therefore excluded from both the bond and equity markets.\footnote{The borrowing constraint is binding for bonds; the young may not borrow by shorting bonds. They are, however, allowed to short equity, but the short-sale equity constraint is non-binding for the set of parameters chosen to calibrate the model; i.e., the young choose not to short equity.}

The middle-aged consumers receive a random wage income $w^1_{t+1}$ in period $t + 1$, and at this time the wage uncertainty is largely resolved for this age cohort. These consumers participate in the financial sector, investing in bond and equity. In the third period, $t + 2$, the consumer has zero wage income and consumes proceeds from financial assets accumulated in previous periods. As such, asset prices (both equity and bonds) are exclusively priced by the middle-aged agents.\footnote{The wage income process of the middle-aged consumer is exogenous in order to avoid modeling the labor-leisure trade-off.}

A consumer born in period $t$ holds $x^b_{t,0} = 0$ bond shares and $x^e_{t,0} = 0$ equity shares. When middle-aged, he holds $x^b_{t,1}$ shares of bond and $x^e_{t,1}$ shares of equity. The old consumers sell their entire bond and stock holdings and consume the proceeds; i.e., $x^b_{t,2} = 0$, and $x^e_{t,2} = 0$. Market clearing in period $t$ requires that the demand for bonds and equity by the young and middle-aged consumers equal their fixed supply:

$$x^b_{t,0} + x^b_{t-1,1} = x^b_{t-1,1} = b, \quad (1a)$$

and similarly,

$$x^e_{t,0} + x^e_{t-1,1} = x^e_{t-1,1} = 1. \quad (1b)$$

Let $c_{t,j}$ denote the consumptions in period $t + j$ ($j = 0, 1, 2$) of a consumer born in period $t$. The budget constraint of the consumer born in period $t$ is:

$$c_{t,0} \leq w^0 \quad (2a)$$

when young,

$$c_{t,1} + x^b_{t,1}p^b_{t+1} + x^e_{t,1}p^e_{t+1} \leq w^1_{t+1} \quad (2b)$$
when middle-aged, and
\[ c_{t,2} \leq s_{t,1}^{b_{t+2}}(p_{t+2}^{b_{t+2}} + 1) + x_{t,1}^e(p_{t+2}^{e_{t+2}} + d_{t+2}) \] (2c)
when old. Furthermore, we require that \( c_{t,0} \geq 0, c_{t,1} \geq 0, \) and \( c_{t,2} \geq 0, \) thus ruling out negative consumption and personal bankruptcy.

We model the joint process of aggregate income and wages of the middle-aged, \((y_t, w_{t}^{1})\), as a time-stationary probability distribution where the aggregate income \( y_t \) is:
\[ y_t = w^0 + w_{t}^{1} + b + d_{t}. \] (3)

In the calibration, \( y_t \) and \( w_{t}^{1} \) assume two values each: \( y_1, y_2 \) and \( w_{1}^{1}, w_{2}^{1} \). These form four states denoted by \( s_t = j \), where \( j = 1, ..., 4 \). The \( 4 \times 4 \) transition probability matrix is denoted by \( \Pi \).

The novelty of this paper lies in introducing increasing risk aversion in this setting. Specifically, the consumer born in period \( t \) has utility:
\[ E \left( \sum_{i=0}^{2} \beta^i u(c_{t,i}, \alpha_i) | I_t \right), \] (4a)
where \( I_t \) is the set of all the information available in period \( t \), and the period utility function is given by:
\[ u(c_{t,i}, \alpha_i) = \frac{c_{t,i}^{1-\alpha_i} - 1}{1 - \alpha_i}, \] (4b)
where \( \alpha_i > 0 \) is the risk aversion parameter. Consistent with the empirical literature, \( \alpha_i \) is assumed to vary with the consumer’s age: the older consumers exhibit higher risk aversion than the younger ones.

In this borrowing-constrained economy there exists a rational expectations equilibrium in which the young do not participate in the bond and equity markets.\(^9\)

\(^9\)Equilibrium is defined as the set of consumption and investment policies of the consumers born in each period and the bond and stock prices in all periods such that: (i) each consumer maximizes his expected utility taking the price processes as given; (ii) bond and equity markets clear in all periods.
Given the consumption constraints ((2a)-(2c)) and the market clearing conditions ((1a)-(1c)), the optimization problem yields the following first order conditions:

\[ u'(c_1) p^b(j) = \beta \sum_{k=1}^{4} (u'(c_2) \ast \{ p^b(k) + 1 \}) \Pi_{jk} \]  

(5a)

and

\[ u'(c_1) p^e(j) = \beta \sum_{k=1}^{4} (u'(c_2) \ast \{ p^e(k) + d(k) \}) \Pi_{jk}, \]  

(5b)

The share of the total wealth saved and invested by the middle-aged investor is given by

\[ \Phi^{s}_{t,1} = \frac{x_{t,1}^{b} p_{t+1}^{b} + x_{t,1}^{e} p_{t+1}^{e}}{w_{t+1}^{1} + x_{t,0}^{b} (p_{t+1}^{b} + 1) + x_{t,0}^{e} (p_{t+1}^{e} + d_{t+1})}, \]  

(6a)

while the relative shares of the wealth invested in bonds and equity are

\[ \Phi^{b}_{t,1} = \frac{x_{t,1}^{b} p_{t+1}^{b}}{w_{t+1}^{1} + x_{t,0}^{b} (p_{t+1}^{b} + 1) + x_{t,0}^{e} (p_{t+1}^{e} + d_{t+1})} \]  

(6b)

and

\[ \Phi^{e}_{t,1} = \frac{x_{t,1}^{e} p_{t+1}^{e}}{w_{t+1}^{1} + x_{t,0}^{b} (p_{t+1}^{b} + 1) + x_{t,0}^{e} (p_{t+1}^{e} + d_{t+1})}, \]  

(6c)

respectively. Additionally, the shares of the savings/investment that goes into bonds and equity are

\[ \omega^{b}_{t,1} = \frac{x_{t,1}^{b} p_{t+1}^{b}}{x_{t,1}^{b} p_{t+1}^{b} + x_{t,1}^{e} p_{t+1}^{e}} \]  

(7a)

and

\[ \omega^{e}_{t,1} = \frac{x_{t,1}^{e} p_{t+1}^{e}}{x_{t,1}^{b} p_{t+1}^{b} + x_{t,1}^{e} p_{t+1}^{e}}, \]  

(7b)

respectively. \( \omega^{b}_{t,1} \) and \( \omega^{e}_{t,1} \) reflect the investor’s choice between the safe and the risky asset as he forms his portfolio.

### 3. Calibration

In order to focus exclusively on the impact of increasing risk aversion on security returns and portfolio choice, we use the same calibration parameters as Constantinides,
Donaldson, and Mehra (2002). As such, we limit our discussion here to a summary of
the key parameters.\footnote{As argued in Constantinides, Donaldson, and Mehra (2002), there are some difficulties in empirically estimating the necessary moment conditions to calibrate the model, mostly because we are dealing with twenty year aggregates in the context of only a century-long data set. Therefore, the standard errors of the point estimates are large resulting in imprecise calibration. See Constantinides, Donaldson, and Mehra (2002) for a more detailed discussion on these moment conditions.}

It should be noted that in this three-period overlapping generations framework, one period - which spans one generation - is assumed to represent 20 years. With this (20-year one-period) set-up, the subjective discount factor ($\beta$) is set equal to 0.44 implying an annual discount factor of 0.96 - the standard annual discount factor used in the literature.

The equilibrium joint distribution of the bond and equity returns depends on a set of parameters which are calibrated as in CDM (2002) based on historical observations and empirical studies. The key calibrated parameters are: 1) The average share of income going to labor ($E(w^1 + w^0)/E(y)$) is set at 0.65; 2) The average share of income going to the labor of the young, $w^0/E(y)$, is set at 0.19 - small enough to ensure that the borrowing constraint is binding in this economy; 3) The average share of income going to interest on government debt, $b/E(y)$, is set at 0.03; 4) The coefficient of variation of the twenty-year wage income of the middle-aged, $\sigma(w^1)/E(w^1)$, is fixed at 0.25; 5) The coefficient of variation of the twenty-year aggregate income, $\sigma(y)/E(y)$, is set at 0.20; 6) The twenty-year auto-correlations and cross-correlation of the labor income of the middle-aged and the aggregate income ($\text{corr}(w^1_t, w^1_{t-1})$, $\text{corr}(y_t, y_{t-1})$, and $\text{corr}(y_t, w^1_t)$) are set at 0.1.\footnote{We also generated results for the following correlation pairs: $\text{corr}(y_t, w^1_t) = 0.1$ and $\text{corr}(w^1_t, w^1_{t-1}) = \text{corr}(y_t, y_{t-1}) = 0.8$, $\text{corr}(y_t, w^1_t) = 0.8$ and $\text{corr}(w^1_t, w^1_{t-1}) = \text{corr}(y_t, y_{t-1}) = 0.1$, and $\text{corr}(y_t, w^1_t) = 0.8$ and $\text{corr}(w^1_t, w^1_{t-1}) = \text{corr}(y_t, y_{t-1}) = 0.8$. Results from these calibrations were similar to the baseline case with $\text{corr}(y_t, w^2_t) = 0.1$ and $\text{corr}(w^1_t, w^1_{t-1}) = \text{corr}(y_t, y_{t-1}) = 0.1$. They are suppressed for brevity and are available upon request.}

The transition matrix of the joint Markov process on the wage income of the middle-
aged consumers and the aggregate income is given by:

\[
\begin{pmatrix}
(y_1, w_1^1) & (y_1, w_1^2) & (y_2, w_2^1) & (y_2, w_2^2) \\
\phi + \Delta & \phi - \Delta & H & \sigma \\
\sigma & H & \phi - \Delta & \pi + \Delta \\
H & \sigma & \pi & \phi
\end{pmatrix},
\tag{8}
\]

where

\[
\phi + \pi + \sigma + H = 1. \tag{9}
\]

Nine parameters need to be estimated: \(y_1/E(y), y_2/E(y), w_1^1/E(y), w_2^1/E(y), \phi, \pi, \sigma, H, \) and \(\Delta\). These are chosen to satisfy the six moment conditions described above as well as condition (9) while requiring that all matrix entries are positive.

Table 1, from the work of Constantinides, Donaldson, and Mehra (2002), shows the historical mean and standard deviations of the annualized, twenty-year holding-period return on the S&P 500 total return series and on the Ibbotson US Government Treasury Long-Term bond yield. Real returns are CPI adjusted. The annualized mean return (for both the equity and bond) is defined as the sample mean of the \([\log\{20\text{-year holding period return}\}]\)/20. The annualized standard deviation of the equity (or bond) return is defined as the sample standard deviation of the \(\log\{20\text{-year holding period return}\}/\sqrt{20}\). The annualized mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of the \(\log\{20\text{-year nominal equity return}\}-\log\{20\text{-year nominal bond return}\}/\sqrt{20}\). From Table 1, the real mean equity return is 6-7% with a standard deviation of 14-16%; the mean bond real return is about 1% with a standard deviation of 7%; and the mean equity premium is 5-7% with a standard deviation of 14-15%.

4. Results

The effect of increasing risk aversion on security returns, equity premium, savings, and portfolio shares are presented in Tables 2-7. Results are reported for various
combinations of middle aged and old agents risk aversion parameters: \( \{ \alpha_1, \alpha_2 \} \) range from \( \{2.00, 2.00\} \) to \( \{6.00, 6.25\} \).\(^{12}\) To highlight the role of IRRA, for each selected pair, we present results both for the case when risk aversion remains constant (e.g., \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.00\} \)) and when it increases with age (e.g., \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.25\} \)).

As discussed in the previous section, our model economies are calibrated for the following set of parameters: \( \sigma(y)/E(y) = 0.20 \), \( \sigma(w^1)/E(w^1) = 0.25 \), \( \text{corr}(y_t, w^1_t) = 0.1 \) and \( \text{corr}(w^1_t, w^1_{t-1}) = \text{corr}(y_t, y_{t-1}) = 0.1 \).

As a preliminary step, we take a brief look at security returns under CRRA as the average level of risk aversion in the economy increases, i.e., as we move from the risk-aversion pair \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.00\} \) to \( \{ \alpha_1, \alpha_2 \} = \{6.00, 6.00\} \) (Table 2, columns (i), (iii), (v), (vi) and (ix)). Consistent with theory, as the overall level of risk aversion increases, equity returns rise, bond returns decline, and equity premium increases. Specifically, the equity premium increases from 2.13% when \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.00\} \) to 4.67% when \( \{ \alpha_1, \alpha_2 \} = \{6.00, 6.00\} \). This is in line with expectations: more risk averse investors generally require a higher premium to hold risky assets. At the same time, a higher average risk aversion also implies a higher demand for bonds, which in turn suppresses equilibrium bond returns. The end result is an increase in equity premium and an increase in bond holdings in the financial portfolio.

Next, we focus on the key innovation: the introduction of IRRA and its impact on security returns and on the equity premium. We find that for each selected pair (i.e., \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.00\} \) vs \( \{ \alpha_1, \alpha_2 \} = \{2.00, 2.25\} \)), a small increase in risk aversion leads to an increase in both equity and bond returns, and a higher equity premium. Overall, the model delivers equity premium values that are consistent with their historical averages without assuming implausibly high levels of risk aversion.\(^{13}\) In our specifi-

\(^{12}\)We do not consider the risk aversion of the young since they are effectively shut out of the securities market. The model introduces some form of limited participation since agents participate in the market in two out of the three period – as savers when middle aged and as dissavers when old.

\(^{13}\)An undesirable feature of the model is that it produces equity returns, bond returns and standard deviations which are higher than the historical averages. IRRA increases both equity and bond returns with the latter exacerbating the risk-free puzzle.
cation, old consumers more risk averse than middle-aged ones \((\alpha_2 > \alpha_1)\) which leads to an increase in both equity and bond returns relative to the baseline case \((\alpha_1 = \alpha_2)\), but the increase in the equity return is larger resulting in an increase in the mean equity premium. For example, equity return is 4.7% higher and the bond return is 2.3% higher with \(\{\alpha_1, \alpha_2\} = \{4.00, 4.25\}\) compared to \(\{\alpha_1, \alpha_2\} = \{4.00, 4.00\}\) (Table 2, columns (v) and (vi)).

The intuition for these results is fairly straightforward: with IRRA, the agents become even more averse to gambles that play out in the future (when old) so they save less and consume more compared to the CRRA scenario. A lower level of savings means that the overall wealth invested in financial market (both in equities and bonds) also declines. On balance, the effect is to increase both equity and bond returns while increasing the equity risk premium.

The introduction of IRRA leads to an increase in the bond return. Middle-aged agents, who will be more risk averse in the future, are now even less willing to give up current consumption for future consumption. Therefore, they demand higher return on the bond (and equity). This can be seen by analyzing the stochastic discount factor (SDF) in this economy:

\[
m_{t+1} = \beta \frac{u'(c_{t+2})}{u'(c_{t+1})} = \beta \left( \frac{c_{t+2}}{c_{t+1}} \right)^{-\alpha_1} \cdot c_{t+2}^{(\alpha_1 - \alpha_2)}.
\]

In the CRRA case when \(\alpha_1 = \alpha_2\), the SDF is \(\beta \left( \frac{c_{t+2}}{c_{t+1}} \right)^{-\alpha_1}\). Thus, the presence of increasing risk aversion \((\alpha_1 < \alpha_2)\) introduces an additional factor, \(c_{t+2}^{(\alpha_1 - \alpha_2)}\), which decreases the standard SDF: the individual agent’s willingness to shift consumption between middle-age and old-age declines, and the risk-free rate (the inverse of \(E(m)\)) increases.\(^{14}\) In a sense, IRRA and the borrowing constraint have opposite effects on bond returns: while borrowing constraint reduces bond returns (the young cannot borrow at

\(^{14}\)The stochastic discount factor depends on the absolute level of consumption; therefore the model is not insensitive to scale. However, the scaling affect is relatively small in our specification. For example, if we double the average level of income in the economy \((y)\), the equity return goes up by 0.58 percent, the bond return by 0.40 percent, and the premium over the bond by 0.20 percent.
the risk-free rate to invest in equity), IRRA raises bond returns (through its impact on the SDF). For example, the SDF is lower when \( \{\alpha_1, \alpha_2\} = \{2.00, 2.25\} \) than when \( \{\alpha_1, \alpha_2\} = \{2.00, 2.00\} \).

Equity returns increase both because the level of savings/investment declines and because the middle-aged cohort know that they will become more risk averse when old and require a much higher equity return (relative to the risk-free rate) in order to invest in equity given its uncertain future payoffs. Thus, in the case of equity, the IRRA reinforces the effect of the borrowing constraint and produces higher equity returns in equilibrium.\(^{15}\)

It should be noted that the higher equity premium is obtained by fairly small differences in risk aversion values – for all cases presented \( \alpha_2 \) is only 0.25 higher than \( \alpha_1 \). More importantly, we find that results are driven primarily by the relative difference between the two risk aversion parameters (how much more risk averse old agents are relative to the middle aged) rather than the \textit{average} risk aversion in the economy (how much more risk averse \textit{both} cohorts are). Table 3 highlights this. As seen, \( \{\alpha_1, \alpha_2\} = \{2.00, 2.25\} \) delivers higher equity and bond returns compared to \( \{\alpha_1, \alpha_2\} = \{6.00, 6.25\} \) despite the fact that the \textit{average} level of risk aversion in the economy is much higher in the second case. This is so because the \textit{relative increase} in risk aversion is higher under the \( \{\alpha_1, \alpha_2\} = \{2.00, 2.25\} \) scenario (12.5%) relative to \( \{\alpha_1, \alpha_2\} = \{6.00, 6.25\} \) case (4.10%). Note however that the equity premium with \( \{\alpha_1, \alpha_2\} = \{6.00, 6.25\} \) is higher. In a more extreme case when risk aversion increases substantially more (\( \{\alpha_1, \alpha_2\} = \{2.00, 2.50\} \)), the pair produces higher equity, bond and \textit{equity premium} than \( \{\alpha_1, \alpha_2\} = \{6.00, 6.25\} \). This suggests while the average level of risk aversion in the economy matters, it is how much more risk averse the agents become as they age.

\(^{15}\)As in CDM (2002), the borrowing constraint increases equity returns because equity returns are exclusively driven by middle-aged agents since the young cannot participate in the market. However, equity does not have the same appeal for the middle-aged as it does for the young because there is no wage uncertainty for the middle-aged. Consumption for this age cohort is highly correlated with equity income which means that equity no longer serves as a hedge against consumption and it requires a higher rate of return for this group.
that drives the key results.

The introduction of IRRA in an OLG model has significant implication for consumption/saving and portfolio shares. First, to analyze the consumption/saving effect, it helps to present the consumption pattern of each age group and the savings/investment of the middle-aged in all states of the economy. These results are summarized in Table 4. The top panel \( \{\alpha_1, \alpha_2\} = \{4.00, 4.00\} \) illustrates why bonds are attractive despite high equity returns. The consumption of the old age cohort is quite variable, leading the middle-aged to invest some of their wealth in bonds since bonds are a hedge against future consumption variability. The young agents’ consumption is the same across all states of the economy (since they simply consume their endowment), while the middle-aged have a relatively smooth consumption pattern.

The bottom panel of Table 4 shows the consumption patterns and middle-aged savings under IRRA. As discussed above, middle-aged investors are now even less willing to give up some of their current consumption in return for higher future consumption. In fact, they consume more today and save less for the future despite higher bond and equity returns.\(^ {16} \) Note also that the variance of middle-aged consumption is much higher under IRRA relative to the CRRA case.

The results for consumptions/savings for various combinations of risk aversion pairs are presented in Table 5. Assuming CRRA and moving from low levels of risk aversion towards higher values, we find that as the average risk aversion in the economy increases (for both middle-aged and old consumers) the level of savings/investment increases modestly. Specifically, two opposing forces determine the level of savings/investment: while more risk averse agents optimally prefer to invest less in risky assets they are also more prudent and accumulate more wealth over the life cycle. Therefore, under the constant risk aversion scenario, the wealth effect dominates the risk aversion effect.

\(^ {16} \)These results are likely to reflect an intertemporal substitution effect rather than a risk aversion effect. DaSilva and Farka (2011) investigate this and explore the impact of IRRA on security returns, savings, and portfolio shares in a model economy with recursive preferences (Epstein-Zin).
These results change dramatically when increasing risk aversion is introduced. For example, the level of savings declines almost by half when old-age risk aversion increases slightly: from $12,794 when \( \alpha_1, \alpha_2 \) = \{5.00, 5.00\} to $6,795 when \( \alpha_1, \alpha_2 \) = \{5.00, 5.25\}. In addition, with IRRA, the lowest level of saving is found for \( \alpha_1, \alpha_2 \) = \{2.00, 2.25\} and the highest for \( \alpha_1, \alpha_2 \) = \{6.00, 6.25\} consistent with the view that in the first scenario, agents become "relatively more" risk averse as they age compared to the second case.

Table 6 shows the portfolio decisions of the (middle-aged) investor for different pairs of risk aversion. It summarizes the share of the wealth saved and invested, the share of the savings/investment that goes into equity, and the share of the savings/investment that goes into bonds. Under CRRA an increase in the average level of risk aversion in the economy leads to a small increase in the level of savings, an increase in bond holdings (\( \Phi^b \)), and a decrease in equity holdings (\( \Phi^e \)). For example, as risk aversion increases from \( \alpha_1, \alpha_2 \) = \{2.00, 2.00\} to \( \alpha_1, \alpha_2 \) = \{6.00, 6.00\}, savings/investment increase from $12,362 (27.7% of total wealth) to $13,060 (29.2% of total wealth), the share of wealth invested in bonds increases from 6% to 12.6%, while the share of wealth invested in equity drops from 21.6% to 16.7%. This is in line with empirical evidence which shows that a higher level of risk aversion increases the demand for bonds (safer asset) and reduces the demand for equity.

The introduction of increasing risk aversion has a significant impact on portfolio shares (Table 6). Focusing on comparable pairs (i.e., \( \alpha_1, \alpha_2 \) = \{4.00, 4.00\} vs \( \alpha_1, \alpha_2 \) = \{4.00, 4.25\}), the overall share of financial portfolio declines with increasing risk aversion as investors save less and consume more. For example, the share of wealth saved and invested declines from 28.0% with \( \alpha_1, \alpha_2 \) = \{4.00, 4.00\} to 12.7% with \( \alpha_1, \alpha_2 \) = \{4.00, 4.25\}. However, both the share of wealth invested in bonds and the share of wealth invested in equity decline, with the decrease in equity share exceeding the bonds'. For example, the share of wealth invested in equity declines from
17.5% to 5.7% while the bonds’ share decreases from 10.5% to around 7.0% (going from {4.00, 4.00} to {4.00, 4.25}).

These findings suggest that with IRRA not only does the overall share of wealth in financial assets decline, but the composition of the financial portfolio also changes as wealth is shifted away from the risky and into the safer asset. Out of his savings, the middle aged investor invests only 44.8% in equity with IRRA ({4.00, 4.25}) compared to 62.4% in the benchmark case ({4.00, 4.00}) (Table 7). Alternatively, the portfolio share of bonds increases from 37.6% to 55.2%. This reflects the desire to move away from risky assets and into safer ones as the agents become more risk averse with age. Recall that the bond is appealing to the investor, despite its lower return, because it works as a hedge against future consumption variability. With increasing risk aversion, the appeal of the bond as a hedge against future consumption variation increases which means that agents invest a larger share of savings in bonds.

It should be noted that the introduction of IRRA produces more realistic portfolio shares compared to CRRA, for each pair of risk aversion parameters. Empirical evidence has consistently found that the share of the risky asset in the portfolio is around 50% (Poterba and Samwick (2001), Bertraut and Starr-McCluer (2002), Ameriks and Zeldes (2004)). With CRRA, the portfolio share of equity in our model ranges from 57%-78%, depending on the average level of risk aversion in the economy (Table 7), which exceeds empirical estimates. When IRRA is introduced, equity shares appear more in line with the empirical evidence, ranging from 45%-53%. It is also important to stress that although IRRA depresses the level of savings/investment and reallocates wealth away from equity and into bonds, our results do not bear out the “market meltdown” hypothesis associated with an ageing population. Even if we assume that risk aversion increases substantially more from middle-aged to old (e.g., from {4.00, 4.00} to {4.00, 4.50}), the share invested in risky asset is still around 27% (Table 7, column (viii)).
In sum, the model with increasing risk aversion produces higher security returns (equity and bond) and standard deviations. Equity return increases significantly more than bond returns, thus easily matching the US equity premium. The overall level of savings/investment in the economy decreases and the composition of the financial portfolio also changes as more wealth is shifted away from the risky and into the safer asset (despite the higher equity returns).

5. Conclusions

This paper suggests a new approach to asset pricing and portfolio allocation, one that takes into consideration the vast body of empirical research linking the agents’ level of risk aversion coefficient to their age. We incorporate increasing risk aversion (IRRA) in the three-period overlapping generations exchange economy of Constantinides, Donaldson, and Mehra (CDM) (2002) with borrowing constraints. We highlight the effect of IRRA by assuming that older agents are more risk averse than middle-aged ones while retaining key features of the CDM (2002) framework. This specification produces interesting and significant results that are in general consistent with US data without assuming unreasonable levels of risk aversion.

We find that an increase in risk aversion leads to an increase in equity and bond returns. However, equity returns increase by considerably more than bond returns, which produces a higher equity premium. IRRA drives the behavior of asset returns: agents become more risk averse towards gambles that play out in the future which means that they consume more and save/invest less. A lower level of savings implies that the overall wealth invested in financial market (both in equity and bonds) also declines, which drives up both equity and bond returns. IRRA has an additional second order effect on equity returns because the middle-aged agents, who know they will become more risk averse when they age, require a higher equity return (relative to the risk-free rate) in order to invest in the risky asset given its uncertain future payoffs.
The introduction of IRRA has also significant implication for consumption/saving and portfolio shares. We find that, with IRRA, the overall share of wealth invested in both equity and bonds declines with the decrease in equity investment exceeding the decline in bond investment. This suggests that IRRA reduces not only the overall share of wealth in the financial assets, but tilts the composition of the financial portfolio as wealth is shifted away from the risky asset and into the safer one. In addition, IRRA delivers portfolio shares of the risky asset that match the US data: the equity share in the portfolio is in the range of 45%-53%. Our results do not support the “market meltdown” hypothesis associated with an ageing population and the retirement of the baby-boomers.

Our findings are driven by fairly small differences in risk aversion values and we consistently assume that risk aversion increases only modestly with age. More importantly, we find that our results are primarily driven by the relative difference between the two risk aversion parameters (how much more risk averse old agents are relative to the middle aged) rather than the average risk aversion in the economy (how much more risk averse both cohorts are).

This study looks at the effect of increasing risk aversion within the context of a simple OLG framework and finds that it has important implications for security returns and portfolio allocations. Nonetheless, the model abstracts from some features that may enrich its results. One interesting generalization would be to separate the effect of increasing risk aversion from the intertemporal rate of substitution. Alternatively, the relaxation of the borrowing constraint may highlight more fully the role of increasing risk aversion on the level of savings, security returns, and household portfolio behavior.
References


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### Table 1
Historical U.S. Real Returns

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<td>Bond</td>
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<tr>
<td>Standard Deviation</td>
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Notes

This table is a replica of Table 1 in Constantinides, Donaldson, and Mehra (2002). It shows the mean and standard deviations of the annualized, twenty-year holding-period return on the S&P 500 total return series and on the Ibbotson U.S. Government Treasury Long-Term bond yield. Real returns are CPI adjusted. The annualized mean return (for both the equity and bond) is defined as the sample mean of the $\log(20\text{-year holding period return})/20$. The annualized standard deviation of the equity (or bond) return is defined as the sample standard deviation of the $\log(20\text{-year holding period return})/\sqrt{20}$. The annualized mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of the $((\log(20\text{-year nominal equity return})-\log(20\text{-year nominal bond return}))/\sqrt{20})$. 
Table 2
Security Returns and Equity Premium with Different Risk Aversion Coefficients

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<td>25.71</td>
<td>30.43</td>
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Notes
This table presents the effects of risk aversion on security returns and on the equity premium. Results are reported for CRRA and IRRA. Results are derived from the following calibration parameters: $\sigma(y)/E(y) = 0.20$, $\sigma(w^i)/E(w^i) = 0.25$, $\text{corr}(y,w^i) = 0.1$, and $\text{corr}(w^i,w^i) = \text{corr}(y,y) = 0.1$. The long-run probabilities for this economy are: $P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$ and $P_4 = 0.275$. 
### Table 3
Relationship between Security Returns/Equity Premium and the Relative Difference in Risk Aversion Coefficients

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<td>6.86</td>
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**Notes**
This table shows the impact of the relative difference in risk aversion parameters on security returns and on the equity premium. Results are derived from the following calibration parameters:

$\sigma(y)/E(y) = 0.20$, $\sigma(w^i)/E(w^i) = 0.25$, $corr(y,w^i) = 0.1$, and $corr(w^j,w^j) = corr(y,y) = 0.1$.

The long-run probabilities for this economy are: $P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$ and $P_4 = 0.275$. 
Table 4
Consumption and Savings/Investments Across Different States
(CRRA and IRRA)

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<th>State 3</th>
<th>State 4</th>
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<td></td>
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<td>32,137</td>
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<tr>
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<td>3.99</td>
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<td><strong>1.85</strong></td>
<td><strong>3.96</strong></td>
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<table>
<thead>
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<th>State 3</th>
<th>State 4</th>
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</table>

Notes
This table shows the behavior of consumption, savings, equity/bond investment, and security returns across all four states with CRRA (panel a) and IRRA (panel b). Results are derived from the following calibration parameters: $\sigma(y)/E(y) = 0.20$, $\sigma(w^1)/E(w^1) = 0.25$, $corr(y, w^1) = 0.1$, and $corr(w^3, w^3) = corr(y, y) = 0.1$. The long-run probabilities for this economy are: $P_1 = 0.275; P_2 = 0.225; P_3 = 0.225$ and $P_4 = 0.275$.  

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Table 5
Savings/Investments with Different Risk Aversion Coefficients

<table>
<thead>
<tr>
<th></th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
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<th>vii</th>
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<tbody>
<tr>
<td>(\alpha_1)</td>
<td>2.00</td>
<td>2.00</td>
<td>3.00</td>
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<tr>
<td>Middle-Aged Consumption</td>
<td>32,288</td>
<td>41,715</td>
<td>32,355</td>
<td>40,238</td>
<td>32,137</td>
<td>38,966</td>
<td>31,856</td>
<td>37,855</td>
<td>31,590</td>
<td>36,893</td>
</tr>
<tr>
<td>Old Consumption</td>
<td>47,111</td>
<td>37,684</td>
<td>47,044</td>
<td>39,161</td>
<td>47,262</td>
<td>40,433</td>
<td>47,543</td>
<td>41,544</td>
<td>47,809</td>
<td>42,506</td>
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<tr>
<td>Young Consumption</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
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</tr>
<tr>
<td>Savings/Investment</td>
<td>12,362</td>
<td>2,935</td>
<td>12,295</td>
<td>4,412</td>
<td>12,513</td>
<td>5,684</td>
<td>12,794</td>
<td>6,795</td>
<td>13,060</td>
<td>7,757</td>
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</table>

Notes
This table presents the effects of risk aversion on the consumption of the three age-cohorts: young, middle-aged and old. It also shows the pattern of savings of the middle-aged with different risk aversion parameters. Results are reported for CRRA and IRRA. Results are derived from the following calibration parameters: \(\sigma(y) / E(y) = 0.20\), \(\sigma(w^1) / E(w^1) = 0.25\), \(corr(y, w^1) = 0.1\), and \(corr(w^1, w^1) = corr(y, y) = 0.1\). The long-run probabilities for this economy are: \(P_1 = 0.275\); \(P_2 = 0.225\); \(P_3 = 0.225\) and \(P_4 = 0.275\).
Table 6  
Portfolio Shares with Different Risk Aversion Coefficients

<table>
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<tbody>
<tr>
<td>$\alpha_1$</td>
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<td>6.00</td>
<td>6.25</td>
</tr>
<tr>
<td>$\varphi^S$</td>
<td>27.69</td>
<td>6.57</td>
<td>27.54</td>
<td>9.88</td>
<td>28.03</td>
<td>12.73</td>
<td>28.65</td>
<td>15.22</td>
<td>29.25</td>
<td>17.37</td>
</tr>
<tr>
<td>$\varphi^B$</td>
<td>6.05</td>
<td>3.09</td>
<td>8.52</td>
<td>5.29</td>
<td>10.53</td>
<td>7.03</td>
<td>11.82</td>
<td>8.34</td>
<td>12.57</td>
<td>9.33</td>
</tr>
<tr>
<td>$\varphi^E$</td>
<td>21.64</td>
<td>3.49</td>
<td>19.02</td>
<td>4.59</td>
<td>17.50</td>
<td>5.70</td>
<td>16.84</td>
<td>6.88</td>
<td>16.68</td>
<td>8.04</td>
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<tr>
<td>Investment in Bond</td>
<td>2,701</td>
<td>1,378</td>
<td>3,805</td>
<td>2,364</td>
<td>4,700</td>
<td>3,138</td>
<td>5,277</td>
<td>3,725</td>
<td>5,611</td>
<td>4,168</td>
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<tr>
<td>Investment in Equity</td>
<td>9,661</td>
<td>1,557</td>
<td>8,490</td>
<td>2,049</td>
<td>7,813</td>
<td>2,546</td>
<td>7,517</td>
<td>3,070</td>
<td>7,449</td>
<td>3,589</td>
</tr>
</tbody>
</table>

Notes  
This table presents the effects of risk aversion on portfolio shares and the total amount invested in equity and bonds. $\varphi^S$ is the share of wealth saved/invested; $\varphi^B$ is the share of wealth invested in bonds; $\varphi^E$ is the share of wealth invested in equity. Results are reported for CRRA and IRRA. Results are derived from the following calibration parameters: $\sigma(y)/E(y) = 0.20$, $\sigma(w^i)/E(w^i) = 0.25$, $corr(y, w^i) = 0.1$, and $corr(w^i, w^j) = corr(y, y) = 0.1$. The long-run probabilities for this economy are: $P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$ and $P_4 = 0.275$.  

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**Table 7**  
*Portfolio Allocation with Different Risk Aversion Coefficients*

<table>
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</tr>
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<tbody>
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<td>5.25</td>
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<td>6.25</td>
</tr>
<tr>
<td>$\omega^B$</td>
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<td>0.47</td>
<td>0.31</td>
<td>0.54</td>
<td>0.38</td>
<td>0.55</td>
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<td>0.41</td>
<td>0.55</td>
<td>0.43</td>
<td>0.54</td>
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<tr>
<td>$\omega^E$</td>
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<td>0.69</td>
<td>0.46</td>
<td>0.62</td>
<td>0.45</td>
<td>27.11</td>
<td>0.59</td>
<td>0.45</td>
<td>0.57</td>
<td>0.46</td>
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<tr>
<td>$\phi^S$</td>
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<td>6.57</td>
<td>27.54</td>
<td>9.88</td>
<td>28.03</td>
<td>12.73</td>
<td>5.36</td>
<td>28.65</td>
<td>15.22</td>
<td>29.25</td>
<td>17.37</td>
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<tr>
<td>$\phi^B$</td>
<td>6.05</td>
<td>3.09</td>
<td>8.52</td>
<td>5.29</td>
<td>10.53</td>
<td>7.03</td>
<td>3.91</td>
<td>11.82</td>
<td>8.34</td>
<td>12.57</td>
<td>9.33</td>
</tr>
<tr>
<td>$\phi^E$</td>
<td>21.64</td>
<td>3.49</td>
<td>19.02</td>
<td>4.59</td>
<td>17.50</td>
<td>5.70</td>
<td>1.45</td>
<td>16.84</td>
<td>6.88</td>
<td>16.68</td>
<td>8.04</td>
</tr>
</tbody>
</table>

Notes  
This table presents the effects of risk aversion on portfolio allocations for bonds and equities. $\omega^B$ is the portfolio share invested in bonds, $\omega^E$ is the portfolio share invested in equity, $\phi^S$ is the share of wealth saved/invested, $\phi^B$ is the share of wealth invested in bonds, and $\phi^E$ is the share of wealth invested in equity. Results are reported for CRRA and IRRA. Results are derived from the following calibration parameters:  
$\sigma(y) / E(y) = 0.20$, $\sigma(w^l) / E(w^l) = 0.25$, $corr(y, w^l) = 0.1$, and $corr(w^l, w^l) = corr(y, y) = 0.1$. The long-run probabilities for this economy are: $P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$ and $P_4 = 0.275$. 
