Learning-by-doing and unemployment dynamics ☆

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ABSTRACT

This paper attempts to assess the impact of skill loss on the persistence of cyclical unemployment. The observations from the Current Population Survey and the Bureau of Labor Statistics suggest a countercyclical total unemployment rate that exhibits high persistence. A framework that features search frictions is developed. Households choose search intensities, and firms create vacancies. Workers accumulate skills through past work experience, or a process of learning-by-doing. This paper extends the learning-by-doing framework to consider endogenous skill loss by the unemployed, or a process of loss-of-learning-by-not-doing. An adverse aggregate technological shock induces workers to reduce their search intensity and firms to reduce their creation of vacancies. As unemployment increases, workers lose their accumulated skills. The skill obsolescence causes a decline in the future marginal productivity of workers. The decline in productivity causes a persistence in the cyclical downturn, and a delay in the recovery of the economy. This allows the model to capture the observed unemployment persistence.

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1. Introduction

The aftermath of the financial crisis and the cyclical downturn that ensued led to an unprecedented length of unemployment durations. Elsby et al. (2011) showed that continued labor market weakness after the crisis led to the highest level of long term unemployment in the postwar period. The persistence of unemployment spells after an economic contraction, and the delay in the economic recovery, has been the focus of several studies.

This paper attempts to assess the impact of skill loss and obsolescence on the persistence of unemployment over the business cycle. The evidence for unemployment persistence is well documented in previous studies. This paper uses aggregated data from the Current Population Survey, and aggregate data from the Bureau of Labor Statistics, to identify the cyclical pattern of unemployment. The observations confirm the previous findings of a countercyclical unemployment that exhibits high persistence over the business cycle.

The paper also develops a model to identify the underlying market interactions that are critical in generating the observed behavior. Endogenous skill acquisition and loss are introduced, in a framework of learning-by-doing. These interactions are captured in a dynamic stochastic general equilibrium model that features search frictions. In this framework, the unemployed choose their search intensity for occupations and firms create vacancies. A standard matching function matches searchers with occupations. Workers accumulate skills through past work experience, or a process of learning-by-doing, similar to the one introduced by Chang et al. (2002). In their work, learning-by-doing is found to provide an important propagation mechanism in real business cycle models. Their framework, however, did not consider equilibrium unemployment. This paper extends their analysis to consider skill loss by the unemployed, or loss-of-learning-by-not-doing. The extension allows the paper to succeed in capturing the observed unemployment persistence. An adverse aggregate technological shock induces workers to reduce their search intensity and firms to reduce their creation of vacancies. As unemployment increases, workers lose their accumulated skills. The skill obsolescence causes a decline in the future marginal productivity of workers. The decline in productivity causes a persistence in the cyclical downturn, and a delay in the recovery of the economy. This allows the model to capture the observed unemployment persistence.

This paper adopts a different approach compared to previous studies that attempted to explain the persistence of unemployment. Some studies considered the aspect of skill loss. For instance, Pissarides (1992) shows that when unemployed workers lose some of their skills, they become less attractive to firms. Thus, firms create less vacancies. The market becomes thin because job seekers have less skills. The thin market leads to more job shortage which in turn perpetuates the thinness. Accordingly, the effect of the shock persists. Ljunqvist and Sargent (1998) formulate a model in which workers' skills depreciate during unemployment spells. After a shock, generous unemployment compensation hinders the process of restructuring the economy. The unemployed lack the incentives to quickly accept new jobs in which skills will have to be accumulated. Consequently, there can be a lengthy

transition with long term unemployment. The fact that welfare benefits are based on past earnings causes the unemployed with depreciated skills to bail out from the labor force.

Esteban-Pretel (2005) and Esteban-Pretel and Faraglia (2010) include the aspect of skill loss if unemployed for an extended period of time in a business cycle model. When the economy suffers an adverse shock, the creation of vacancies declines thus lengthening unemployment spells. The increase in the duration of unemployment causes workers to lose their skills. The increase in the unemployment of the unskilled, who have a lower probability of finding a job, raises the average duration of unemployment in the economy and accordingly the persistence of unemployment.

Khalifa (2012) shows that, in a cyclical downturn, skilled workers compete with unskilled workers over unskilled jobs. Thus, the former crowd out the latter into unemployment. This downgrading of jobs in a cyclical downturn, or the increase in the labor input of the skilled in unskilled occupations, and the subsequent crowding out of the unskilled into unemployment, provide a possible explanation to unemployment persistence. Khalifa (2013) argues that, in a cyclical downturn, there is a mismatch between the educational qualifications of the skilled workers and the educational requirements of the unskilled jobs they occupy. Skilled workers lose their skills not only while unemployed for an extended period of time, but also when employed in occupations that do not require their specific skills. As the mismatched skilled workers lose their skills, and become unskilled, they are more likely to be crowded out of unskilled occupations due to their higher separation rates. These mismatched workers who lose their skills and flow into unemployment as unskilled unemployed, in addition to the unemployed skilled workers who lose their skills and become unskilled unemployed, are less attractive to potential employers. This lowers the probability of employment, and causes unemployment to exhibit persistence.

Pries (2004) argues that even though unemployed workers find jobs quickly, due to the high job finding rate following a shock that triggers a burst of job loss, the newly found jobs often last only a short time. After an initial job loss, a worker may experience several short lived jobs before settling into more stable employment. This recurring job loss contributes to the persistence of unemployment.

Several studies attempted to explain unemployment persistence introducing the aspect of ranking of job applicants. Blanchard and Diamond (1994) argue that when firms receive multiple applications, they hire the worker who has been unemployed for the least amount of time. This leads to the persistence of unemployment spells. Eriksson and Gottfries (2005) argue that employers use information on whether the applicant is employed or unemployed as a hiring criterion, since the perceived productivity of an unemployed worker may be lower than that of an employed worker. This ranking of job applicants by employment status increases the level and persistence of unemployment. Eriksson (2006) extends this framework to argue that long term unemployed workers do not compete well with other job applicants because they lost the abilities that employers find attractive. In a model with short term and long term unemployed workers, firms prefer to hire the unemployed who have not lost their human capital. This ranking of job applicants results in a lengthy adjustment process, and is capable of generating persistence after an adverse shock. Ravenna and Walsh (2012) show that following an adverse shock to the economy, the share of low efficiency workers in the pool of unemployed increases. This decreases the incentive of firms to create vacancies. Therefore, heterogeneity in worker's efficiency amplifies unemployment fluctuations and leads to persistent buildups of unemployment.

This paper, however, argues that unemployment persistence can be reproduced in a model without the aspects of recurring job loss, ranking of job applicants, job competition, or labor mismatch, and improves upon the studies that incorporate skill loss by introducing the aspect of endogenous skill obsolescence through a process of loss-of-learning-by-not-doing.

The paper offers several contributions: First, this is the first study that incorporates the feature of learning-by-doing in an equilibrium unemployment framework. Second, this is the only study that uses the aspect of endogenous skill loss to explain unemployment persistence in a business cycle framework. Third, this is the first study to expand the learning-by-doing framework to include endogenous skill loss, or loss-of-learning-by-not-doing. Fourth, the paper is able to replicate the persistence of unemployment spells after an adverse shock to the economy using these additional features.

The remainder of the paper is organized as follows: Section 2 presents the stylized facts, Section 3 develops the model, Section 4 discusses the calibration, Section 5 analyzes the results, Section 6 includes the sensitivity analysis, Section 7 concludes, and Section 8 includes the data and derivations appendices. References, tables and figures are included thereafter.

2. Observations

To derive the business cycle patterns of unemployment, a time series is compiled from the Outgoing Rotation Group of the Current Population Survey CPS[1]. This Survey provides monthly information from January 1979 until December 2008 on the participants' employment status. To compile a time series out of this survey, the labor market participants in each monthly file are divided into those employed and those unemployed. The proportion of the unemployed out of the total sample is calculated. This monthly time series of the unemployment rate is transformed into quarterly data by taking three months' averages. The data average during the period understudy of the proportion of the employed out of the total labor force is 0.94, which gives an average total unemployment rate of 6%.

The cross correlation coefficients between real gross domestic product in period t and the total unemployment rate in lag and lead periods are displayed in Table 3. These patterns demonstrate that the total unemployment rate is countercyclical where the cross correlation coefficient of −0.8877 is statistically significant. In addition to the aggregated unemployment rate compiled from the Current population Survey CPS, Table 3 also shows the cyclical pattern of the aggregate unemployment rate extracted from the Bureau of Labor Statistics BLS. The pattern shows that the total unemployment rate is countercyclical where the cross correlation coefficient of −0.8834 is statistically significant.

Table 4 displays the serial correlations of the total unemployment rate. The observations from the aggregated CPS data show the high persistence of total unemployment. The persistence of the aggregate unemployment rate from the BLS data is consistent with that of the CPS data. For instance, the first lagged serial correlation is 0.870 in the CPS, and 0.878 in the BLS. The remaining lagged serial correlations are similar in both data sets as well.

3. Model

Consider an economy where time is infinite and discrete. The representative household chooses optimally the search intensity. The representative firm chooses optimally the creation of vacancies. Searchers and vacancies are matched through a standard matching function. Employed workers acquire skills through past work experience, or learning-by-doing. Unemployed workers lose their skills, through a process of loss-of-learning-by-not-doing.

3.1. Households

Let N_t denote the number of the employed workers, and U_t denote the number of the unemployed workers. The labor force is normalized

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[1] Detailed data description is included in Appendix 8.1.

to one. In this context, the household members are divided into those employed and those unemployed as follows

$$U_t = 1 - N_t. \tag{1}$$

Time for all types is normalized to one. An unemployed worker uses a portion S_t of its time to search for an occupation, and $(1 - S_t)$ for leisure. An employed worker spends a portion H_t hours at work and $(1 - H_t)$ for leisure.

As different employment histories amongst members of a household can lead to heterogeneous wealth positions, we follow the literature in assuming that each household is thought of as an extended family whose members perfectly insure each other against variations in labor income due to employment or unemployment. Remaining within the confines of complete markets allows solving the program of a representative household, who chooses consumption and search intensities to maximize the expected discounted infinite sum of its instantaneous utility which is separable in consumption and leisure. Assuming the household has the following value function $\Gamma_t^H = \Gamma^H(X_t, N_t)$, the optimization problem of the household can be written in the following recursive form

$$\Gamma_t^H = \underset{\{C_t, S_t\}}{Max} \left\{ \mho(C_t) + U_t \Omega_t^U + N_t \Omega_t^N + \beta E_t \left[\Gamma_{t+1}^H \right] \right\} \tag{2}$$

where E_t is the expectation operator conditional on the information set available in period t, β is the discount factor and $\mho(C_t)$ is the utility of period t consumption of the household C_t. $\Omega_t^U = \Omega^U(1 - S_t)$ denote the utility of period t leisure of the unemployed. $\Omega_t^N = \Omega^N(1 - H_t)$ denote the utility of period t leisure of the employed. This is subject to the following budget constraint

$$C_t = N_t H_t X_t W_t + D_t \tag{3}$$

where W_t is the period t wage, and D_t is the dividends distributed by firms. Let X_t denote the skill level of labor, which raises the effective unit of labor supplied. The skill accumulates over time according to

$$\ln\left(\frac{X_{t+1}}{X}\right) = \phi \ln\left(\frac{X_t}{X}\right) + [(1-\chi) + P_t]\left[\sigma \ln\left(\frac{H_t}{H}\right)\right] - [\chi + (1-P_t)] \\ \times \left[\delta\phi \ln\left(\frac{X_t}{X}\right)\right] \tag{4}$$

where variables without time subscript denote steady states. Skill accumulation is persistent but not permanent. Skills decay over time at a rate $0 \le \phi < 1$. χ is the separation rate, and P_t is the matching probability. This formulation also implies that an increase in the number of hours worked contributes to an improvement in skills with an elasticity of $\sigma \ge 0$. This occurs with the probability $[(1 - \chi) + P_t]$, that the worker is employed in the current period, either because the worker is not separated from a match or because the worker is matched with an occupation out of unemployment. This formulation for skill accumulation, introduces skill loss by the unemployed at a rate δ. This occurs with the probability $[\chi + (1 - P_t)]$, that the worker is unemployed in the current period, either because the worker is separated from a job or because the unemployed is not matched with an occupation. Thus, the skills' decay is enhanced by the parameter δ in case of unemployment. This equation is considered the main contribution of the paper. It incorporates learning-by-doing into a framework with equilibrium unemployment, and it extends the learning-by-doing analysis to include endogenous skill loss or loss-of-learning-by-not-doing in the case of unemployment.

The households also take into consideration the employment dynamics of the workers. The employed workers in period $t + 1$ are comprised of those who are not exogenously separated in period t according

to the separation rate χ, in addition to the new matches from the searchers pool

$$N_{t+1} = (1 - \chi)N_t + P_t S_t U_t \tag{5}$$

where $P_t = \frac{M_t}{S_t U_t}$ is the probability that a searcher is matched with an occupation, and $M_t = M(V_t, S_t U_t)$ represents the number of matches. The constant separation rates are justified by Hall (2005), who concludes that over the past fifty years separation rates remained almost constant in the United States, and by Shimer (2005) who demonstrates that separation rates exhibit acyclicality. The matching function is homogeneous of degree one function of the number of corresponding vacancies, V_t and effective searchers, $S_t U_t$.

The representative household chooses consumption such that the marginal utility of consumption equals the Lagrange multiplier λ_t

$$\frac{\partial \mho(C_t)}{\partial C_t} = \lambda_t. \tag{6}$$

The household chooses the optimal proportion of time to search for an occupation, S_t, such that the disutility from increasing search by one unit is offset by the discounted expected value of an additional worker employed in an occupation

$$\frac{\partial \Omega^U}{\partial S_t} + \beta P_t E_t \left[\frac{\partial \Gamma_{t+1}^H}{\partial N_{t+1}} \right] = 0. \tag{7}$$

From the envelope theorem, an additional matched worker with an occupation accrue a value to the household that is given by

$$\frac{\partial \Gamma_t^H}{\partial N_t} = -\Omega^U(1 - S_t) + \Omega^N(1 - H_t) + \lambda_t W_t X_t H_t \\ + \beta(1 - \chi - P_t S_t)E_t \left[\frac{\partial \Gamma_{t+1}^H}{\partial N_{t+1}} \right].$$

Substituting the envelope condition into the first order conditions yields the following representative household's optimal condition

$$\frac{\tau^U}{\beta P_t} = -\tau^U E_t(1 - S_{t+1}) + \tau^N E_t(1 - H_{t+1}) + E_t\left[\frac{H_{t+1} W_{t+1} X_{t+1}}{C_{t+1}} \right] \\ + \tau^U E_t\left(\frac{1 - \chi}{P_{t+1}} - S_{t+1} \right) \tag{8}$$

where τ^U is the marginal utility of leisure of the unemployed, and τ^N is the marginal utility of leisure of the employed.

3.2. Firms

The representative firm chooses the number of vacancies to create, in order to maximize the discounted expected infinite sum of its future profit streams. The profit function is given by the difference between the value of its production, where the price of one unit of output is normalized to one, and the total cost incurred for creating the vacancies as well as the labor wage cost. Assuming the firm has the following value function $\Gamma_t^F = \Gamma^F(X_t, N_t)$, the optimization problem can be written in the following recursive form

$$\Gamma_t^F = \underset{\{V_t\}}{Max} \left\{ Y_t - \omega V_t - N_t H_t X_t W_t + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} \Gamma_{t+1}^F \right] \right\} \tag{9}$$

where ω is the cost of creating a vacancy. The discount factor of firms is given such that it effectively evaluates profits in terms of the values attached to them by households, who ultimately own the firms. Thus, the utility based and time varying discount factor used by the firm is

given by $\left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right)$. The maximization is subject to the following production function

$$Y_t = Y[A_t, (H_t N_t X_t)] \tag{10}$$

where A_t is the aggregate technology. The maximization problem of the firm is also subject to the following employment dynamics

$$N_{t+1} = (1 - \chi)N_t + q_t V_t \tag{11}$$

where $q_t = \frac{M_t}{V_t}$ is the probability that a vacancy is filled by a worker. The firm chooses the optimal level of vacancies to post, V_t, such that the expected marginal cost of posting a vacancy is equal to the discounted expected value of creating an occupation from this vacancy

$$\omega = q_t \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}} \right]. \tag{12}$$

From the envelope theorem, the value of an additional worker in an occupation for the firm is given by the difference between its marginal productivity and the wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated

$$\frac{\partial \Gamma_t^F}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} - H_t X_t W_t + (1 - \chi)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}} \right]. \tag{13}$$

Substituting the envelope condition into the first order condition yields the representative firm's optimal condition

$$\frac{\omega}{q_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\partial Y_{t+1}}{\partial N_{t+1}} - H_{t+1} X_{t+1} W_{t+1} + (1 - \chi) \frac{\omega}{q_{t+1}} \right) \right]. \tag{14}$$

3.3. Wages and hours

We follow the literature in assuming that a realized match shares the surplus through a bargaining problem. Therefore, the wage is given by[2]

$$H_t X_t W_t = (1 - \xi)\left[\frac{\partial Y_t}{\partial N_t} + \left(\frac{\omega}{q_t} P_t S_t \right) \right] + \xi C_t \left[\Omega^U (1 - S_t) - \Omega^N (1 - H_t) \right] \tag{15}$$

where ξ is the firm's share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded by a fraction $(1 - \xi)$ of both the firm's revenues from the worker's productivity and the discounted expected value of the match for the firm. The second term indicates that the worker is compensated by a fraction ξ for the foregone benefit from the worker's outside option or the difference between the leisure of an unemployed and that of an employed worker.

The hours of work are chosen such that the disutility of leisure from increasing the hours worked by one unit is offset by the increase in marginal productivity due to an increase in hours by one unit[3]

$$\frac{\partial \left(\frac{\partial Y_t}{\partial N_t} \right)}{\partial H_t} + \left(\frac{1}{\lambda_t} \right) \frac{\partial \Omega_t^N}{\partial H_t} = 0. \tag{16}$$

To close the model[4], we have

$$Y_t = C_t + \omega V_t. \tag{17}$$

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[2] Detailed derivations are included in Appendix 8.2.1.
[3] Detailed derivations are included in Appendix 8.2.2.
[4] All equilibrium conditions and steady state values of variables are available upon request.

4. Calibration

The functional forms are determined and the parameters are calibrated in order to solve the model numerically. In this context, numerical values are assigned to the structural parameters in order to conduct a quantitative analysis. Table 2 shows the values chosen for the parameters of the model. In this context, some of the parameters are set as is standard in the literature. Since information may not be available for the other parameters, their values are computed in the steady state system of equations after setting values for variables quantifiable from the data.

The steady state values for certain variables are calculated from the averages in the dataset during the period under study. For instance, the proportion of the employed type is set at $N = 0.94$ and the unemployed at $U = 0.06$, which are equal to the data averages.

The household's discount factor β is given by 0.98, which is standard in the literature. The instantaneous utility function of consumption is represented by the logarithm of consumption expenditures $\upsilon(C_t) = ln(C_t)$. The instantaneous utility function of leisure is given by $\Omega_t^U = \tau^U(1 - S_t)$ and $\Omega_t^N = \tau^N(1 - H_t)$. The parameter in the utility of leisure for the unemployed τ^U is given by 0.1, and for the employed τ^N is given by 0.6. These parameters are solved for in the steady state equation for the optimal hours of work, given the proportion of employment and hours of work.

The matching function is represented as a Cobb–Douglas specification with constant returns to scale, and is given by $M_t = T(V_t)^\gamma (S_t U_t)^{1-\gamma}$, where $\gamma \in (0, 1)$ is the elasticity of matching with respect to vacancies. The elasticity of matches with respect to vacancies γ is set at 0.5, as is standard in the literature. T is the level parameter which captures all factors that influence the efficiency of matching. The level parameter T is set at 0.4. The choice of the level parameter is determined to target the separation rates. In the steady state, the flows out of employment equal the flows out of unemployment. Thus, we have $\chi N = M$ in the steady state. Therefore, the choice of T determines the matches, and accordingly targets the separation rates. The separation rate χ is given by 0.01, such that it is close to the average separation rate calculated by Hall (2005) and Shimer (2005).

The cost of creating a vacancy ω is given by 2.2. This is determined through the steady state equation of the optimal number of vacancies. The firm's share of the surplus ξ is set at 0.5, as is standard in the literature. The bargaining power of the households are set equal to the elasticity of matching with respect to vacancies, which as shown in Hosios (1990) implies that the bargaining process yields a Pareto optimal allocation of resources.

The parameter in the skill accumulation formulation ϕ is set at 0.797, and the parameter σ is set at 0.111. These are similar to the posterior estimates in the Bayesian estimation in Chang et al. (2002). The probability of skill loss δ is given by 0.25.

Table 2
Calibration of model parameters.

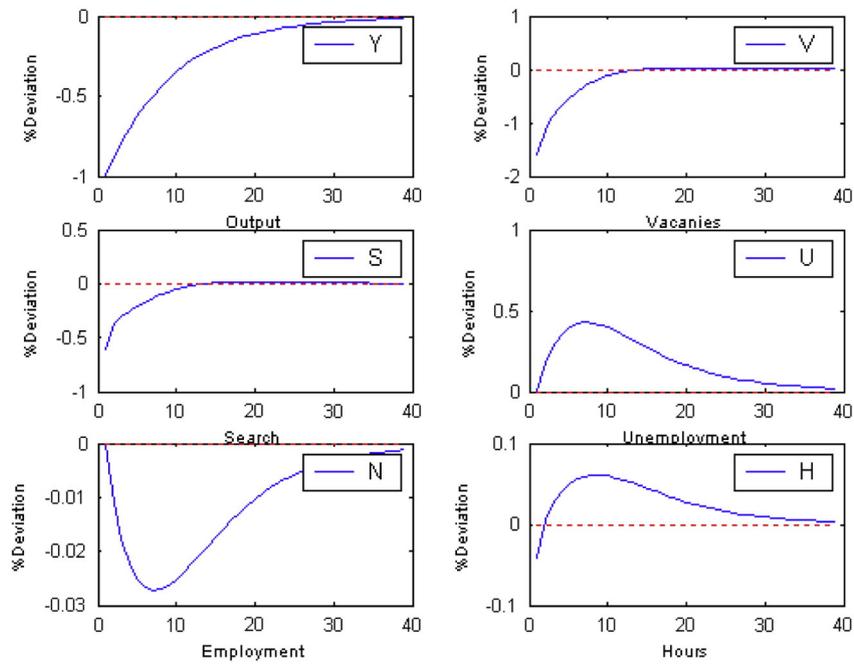| Parameter	Value	Description
β	0.98	Household discount factor
χ	0.01	Separation rate from occupations
γ	0.5	Elasticity of matches with respect to vacancies
μ	0.5	Elasticity in production function
ξ	0.5	Firm share from bargaining with a worker
σ	0.111	Elasticity of hours in skill acquisition
ϕ	0.797	Skill acquisition parameter
δ	0.25	Skill loss parameter
ρ^A	0.9	Autoregressive coefficient of aggregate technology
$\sigma_{\epsilon A}$	0.0049	Standard deviation of the aggregate technology shock
ω	2.28	Cost of posting a vacancy
T	0.4	Efficiency in the matching function
τ^U	0.1	Parameter in the utility of leisure of the unemployed
τ^N	0.6	Parameter in the utility of leisure of the employed

Fig. 1. Model impulse response functions to a negative aggregate technological shock (Learning-by-doing).

The technological constraints faced by the firm is also represented by $Y_t = A_t(H_t N_t X_t)^\mu$, where $\mu \in (0, 1)$ is the elasticity of output. The logarithm of the aggregate technology A_t is assumed to follow an AR(1) process as follows

$$logA_{t+1} = \rho^A logA_t + \epsilon^A_{t+1} \tag{18}$$

where ϵ^A_{t+1} is an independently and identically distributed random variable drawn from a normal distribution with mean zero and standard deviation denoted by $\sigma_{\epsilon A}$. The elasticity parameter in the production function μ is given by 0.5, as in Krause and Lubik (2004). The autoregressive coefficient in the technological law of motion ρ^A is given by 0.9. As is common in the literature, an innovation variance is

chosen such that the baseline model's predictions match the standard deviation of the United States' GDP, which is 1.62%. Consequently, the standard deviation of technology is set to $\sigma_{\epsilon A} = 0.0049$.

5. Analysis

The model is solved by computing the nonstochastic steady state around which the equation system is linearized. The resulting model is solved by the methods developed in Sims (2002). The impulse responses in Figs. 1 and 2 show the dynamic evolution of the variables of interest along with a deviation of output from its long run trend as a consequence of a negative aggregate technological shock.
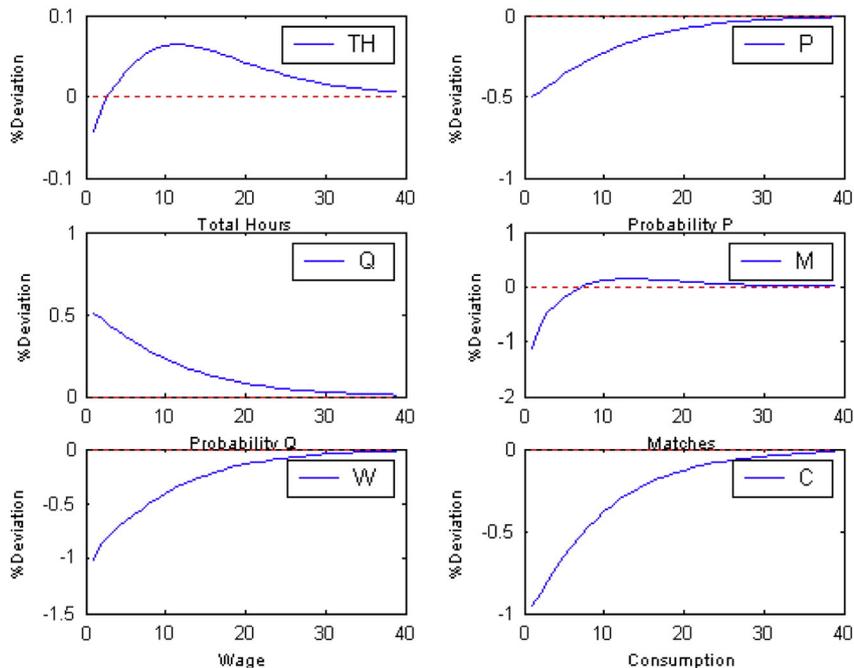

Fig. 2. Model impulse response functions to a negative aggregate technological shock (Learning-by-doing).

Table 3
Data and model moments. Standard errors in () calculated by bootstrapping.

	Cross correlations of output(t) and $U(t + i)$								
U	$U(t - 4)$	$U(t - 3)$	$U(t - 2)$	$U(t - 1)$	$U(t)$	$U(t + 1)$	$U(t + 2)$	$U(t + 3)$	$U(t + 4)$
CPS	−0.1957	−0.3722	−0.5563	−0.7624	−0.8877	−0.8065	−0.6990	−0.5396	−0.3602
	(0.0861)	(0.0813)	(0.0742)	(0.0460)	(0.0359)	(0.0363)	(0.0513)	(0.0750)	(0.0891)
BLS	−0.1571	−0.3422	−0.5419	−0.7514	−0.8834	−0.8505	−0.7265	−0.5477	−0.3328
	(0.0889)	(0.0838)	(0.0724)	(0.0436)	(0.0248)	(0.0314)	(0.0548)	(0.0803)	(0.0972)
Model	−0.6342	−0.6760	−0.7231	−0.7697	−0.8193	−0.8827	−0.9116	−0.9245	−0.9287
	(0.0514)	(0.0457)	(0.0425)	(0.0354)	(0.0320)	(0.0223)	(0.0156)	(0.0141)	(0.0130)

Table 4
Data and model unemployment serial correlations.

	Variable	$\rho(x_t, x_{t-1})$	$\rho(x_t, x_{t-2})$	$\rho(x_t, x_{t-3})$	$\rho(x_t, x_{t-4})$	$\rho(x_t, x_{t-5})$
CPS	U_t	0.870	0.695	0.492	0.299	0.101
BLS	U_t	0.878	0.691	0.480	0.266	0.085
Model (Learning-by-doing)	U_t	0.917	0.833	0.743	0.651	0.558
Model (No learning-by-doing)	U_t	0.867	0.728	0.600	0.488	0.390

The adverse shock decreases the productivity of workers. This reduces the discounted expected value of an additional worker to the firm. The firm posts vacancies such that the expected marginal cost of posting a vacancy is equal to the discounted expected value of creating an occupation from this vacancy. Accordingly, the decrease in the marginal productivity of workers induces firms to decrease their creation of vacancies.

The worker chooses the optimal proportion of time to search for an occupation such that the disutility from increasing search by one unit is offset by the discounted expected value of an additional worker employed in an occupation. The discounted expected value of an additional worker employed declined due to the decline in the wage as a consequence of the decline in the marginal productivity. Accordingly, the unemployed decrease their search intensity for occupations.

The decline in vacancy creation and search intensity, causes a decline in the matches. The decline in the matches leads to a decline in the probabilities of matching as well. This leads to a decrease in the employment and an increase in the unemployment. Unemployment exhibits

persistence, consistently with the observations. The impulse responses show a high persistence of total unemployment. This can be attributed to the process of skill loss. As unemployment increases, workers lose their accumulated skills. Current skill loss affects future labor productivity. This leads to a persistence in the cyclical downturn, which causes a persistence in unemployment.

The hours of work are chosen such that the disutility of leisure from increasing the hours of work by one unit is offset by the increase in the marginal productivity due to an increase in hours by one unit. The impulse response shows that the hours increase after a brief decline. The economy recovery is captured first in the increase in the hours of work, as the marginal productivity increases significantly due to an increase in one hour of work.

Comparing the moments of the model in Table 3 to the data observations, the model succeeds in replicating the countercyclical total unemployment where the correlation coefficient equals −0.8193, which is statistically significant. The success of the model can be also assessed by comparing the serial correlations of the total unemployment rate
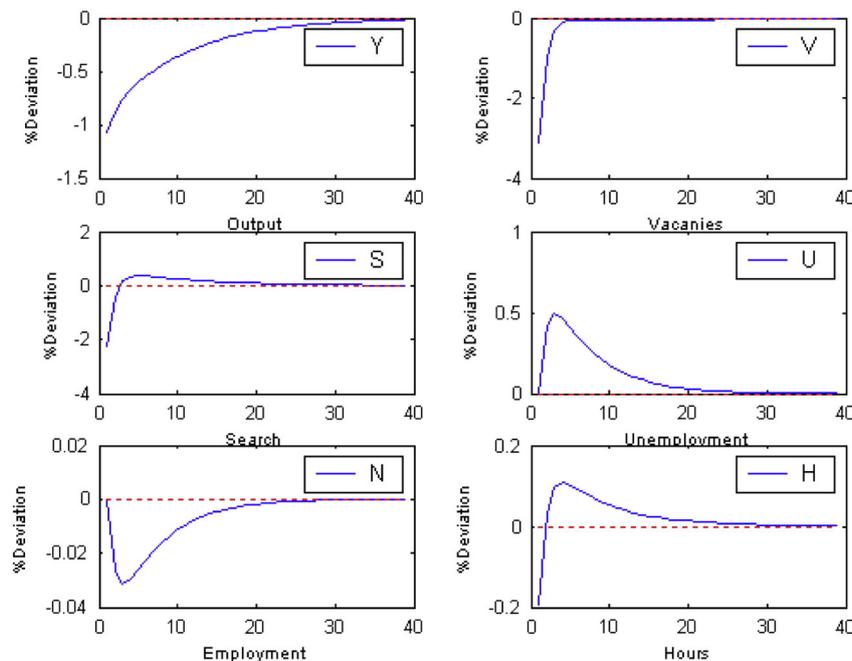
Fig. 3. Model impulse response functions to a negative aggregate technological shock (No learning-by-doing).
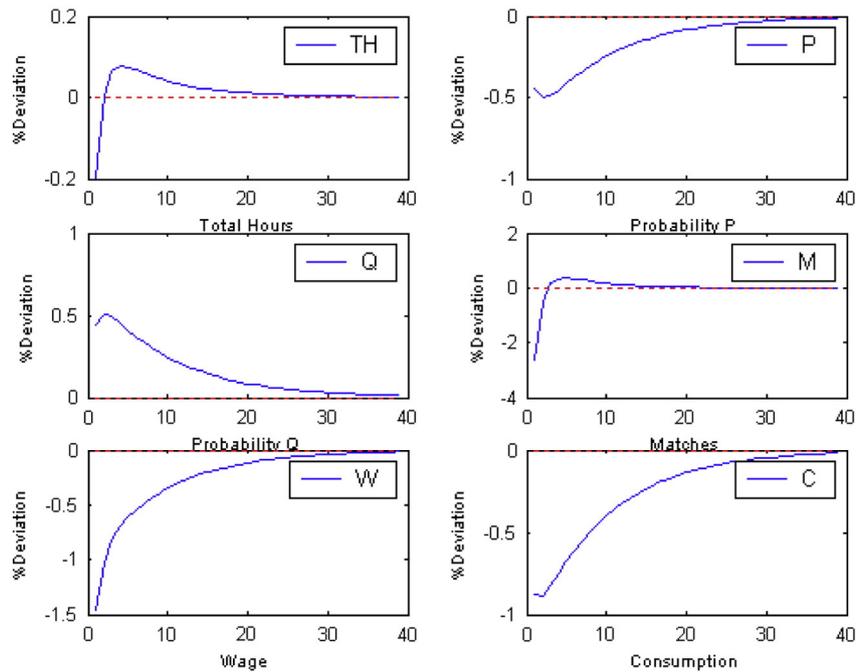
Fig. 4. Model impulse response functions to a negative aggregate technological shock (No learning-by-doing).

produced by the model, and those observed in the data. Table 4 shows that the model succeeds in reproducing the high persistence observed in the data. For instance, the first lag serial correlation of total unemployment is 0.870 in the CPS, 0.878 in the BLS, and 0.917 in the model. For the remaining lagged serial correlations of the unemployment rate, the persistence is higher in the model than that observed in the data.

6. Sensitivity analysis

The robustness of the results of the model is examined to check whether the dynamic evolution of the variables of interest is sensitive to the features of a specific framework. The first robustness test is to compare the results of the benchmark model that includes the aspect of learning-by-doing and another model in which this feature is eliminated. In this model, $\sigma = \phi = \delta = 0$, and is referred to as the "No-learning-by-doing model" hereinafter. The impulse response functions of the "No learning-by-doing model" are included in Figs. 3 and 4. The impulse responses show similar patterns to the ones in the benchmark model. However, it is obvious that unemployment exhibits less persistence in this model than in the benchmark model. This is confirmed with the serial correlations of unemployment in the "No-learning-by-doing model" that are included in Table 4. The serial correlations confirm that the persistence of unemployment spells is higher in the benchmark model than in the "No-learning-by-doing model."

Another robustness test considers the skill loss parameter, δ. The parameter is calibrated at 0.25 in the benchmark model. An increase in the

value of the parameter to 0.5 and 0.75 is also considered. Table 5 shows the serial correlations of unemployment when the skill loss parameter, $\delta = 0.25$, 0.5, and 0.75. The results are robust, and show that the serial correlations do not significantly change with a change in this parameter.

7. Conclusion

This paper attempts to explain the cyclical pattern and persistence of total unemployment. The observations suggest a countercyclical unemployment that exhibits high persistence over the business cycle. The paper develops a model where skill acquisition and loss are introduced endogenously, in a framework of learning-by-doing. These interactions are captured in a dynamic stochastic general equilibrium model that features search frictions. In this framework, the unemployed choose their search intensity. Firms create vacancies, and a standard matching function matches workers with occupations. Workers accumulate skills through past work experience, or a process of learning-by-doing. This paper also considers endogenous skill loss by the unemployed, or loss-of-learning-by-not-doing. The extension allows the paper to succeed in capturing the observed unemployment persistence. An adverse aggregate technological shock induces workers to reduce their search intensity and firms to decrease their posting of vacancies. As unemployment increases, workers lose their accumulated skills. This skill obsolescence causes a decline in the future marginal productivity of workers, causing a persistence in the cyclical downturn. This allows the model to capture the observed unemployment persistence.

Table 5
Model unemployment serial correlations.

	δ	Variable	$\rho(x_t, x_{t-1})$	$\rho(x_t, x_{t-2})$	$\rho(x_t, x_{t-3})$	$\rho(x_t, x_{t-4})$	$\rho(x_t, x_{t-5})$
Model (Learning-by-doing)	0.25	U_t	0.917	0.833	0.743	0.651	0.558
Model (Learning-by-doing)	0.50	U_t	0.914	0.828	0.737	0.645	0.553
Model (Learning-by-doing)	0.75	U_t	0.913	0.826	0.735	0.643	0.550

Appendix

A.1. Data

The data set used is the Outgoing Rotation Group of the Current Population Survey. The Current Population Survey is a rotating panel. After the fourth month in the survey, the participants take an eight month hiatus. Afterwards, they are interviewed for another four months, and after the eighth month in sample, they are completely dropped from the survey. The Outgoing Rotation series is a merged collection of the 4th and 8th month-in-sample groups from all 12 months. These two groups play a special role as they are given additional questions, the answers to which are collected in the Outgoing Rotation Group files. The data is monthly and covers the period from January 1979 until December 2008. At the end of each year, the 12 monthly files from January till December are concatenated into a single annual file. The variables extracted are as follows:

Table 1
Extracted variables.

Variable	Definition
MONTH	Month of interview
MLR	Monthly labor force recode
ERNWGT	Earnings weight

Each annual file is divided into monthly files according to the variable MONTH. For each monthly file, participants in the labor force are split into those employed and those unemployed according to MLR. This variable distinguishes between the employed, the unemployed and those not in the labor force. The unemployment rate is calculated using the proper weights ERNWGT. These weights are created for each month such that, when applied, the resulting counts are representative of the national counts. Thus, the proper application of weights enables the results to be presented in terms of the population of the United States as a whole, instead of just the participants in the survey.

Finally, the Real Gross Domestic Product data (Chained Dollars, seasonally adjusted at annual rates) is extracted from the National Income and Product Accounts NIPA. As the Gross Domestic Product data is quarterly, these monthly time series are transformed into quarterly ones by taking three months averages. The data is seasonally adjusted or deseasonalized using a ratio to moving average multiplicative seasonal filter. All variables are detrended using the Hodrick Prescott filter with a smoothing parameter of 1600.

The aggregate unemployment rate is extracted from the Bureau of Labor Statistics. The data is the monthly seasonally adjusted percentage of unemployment in the labor force of those 16 years and over. The aggregate data is detrended using the Hodrick Prescott filter with a smoothing parameter of 1600.

A.2. Derivations

A.2.1. The wage
The wage is determined by

$$W_t = argmax \left[\frac{1}{\lambda_t}\frac{\partial \Gamma_t^H}{\partial N_t}\right]^{1-\xi} \left[\frac{\partial \Gamma_t^F}{\partial N_t}\right]^{\xi}.$$

Then the sharing rule implies $\xi \left[\frac{\partial \Gamma_t^H}{\partial N_t}\right] = (1-\xi)\lambda_t \left[\frac{\partial \Gamma_t^F}{\partial N_t}\right]$. Substituting the envelope conditions of the household $\frac{\partial \Gamma_t^H}{\partial N_t}$ and of the firm $\frac{\partial \Gamma_t^F}{\partial N_t}$, in addition to

$$\xi \frac{\beta}{\lambda_t} E_t \left[\frac{\partial \Gamma_{t+1}^H}{\partial N_{t+1}}\right] = (1-\xi)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}}\right] = (1-\xi)\frac{\omega}{q_t}$$

from the first order condition yields

$$\xi \left[-\Omega^U(1-S_t) + \Omega^N(1-H_t) + \lambda_t W_t X_t H_t + \beta(1-\chi-P_tS_t)E_t\left(\frac{\partial \Gamma_{t+1}^H}{\partial N_{t+1}}\right)\right]$$
$$= (1-\xi)\lambda_t \left[\frac{\partial Y_t}{\partial N_t} - H_t X_t W_t + (1-\chi)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}}\right]\right].$$

Solving for the equilibrium wage rule yields Eq. (15).

A.2.2. The hours
The hours of work is determined by

$$H_t = argmax \left[\left(\frac{1}{\lambda_t}\frac{\partial \Gamma_t^H}{\partial N_t}\right) + \left(\frac{\partial \Gamma_t^F}{\partial N_t}\right)\right].$$

Substituting the envelope conditions for $\frac{\partial \Gamma_t^H}{\partial N_t}$ and $\frac{\partial \Gamma_t^F}{\partial N_t}$, the hours are thus given by Eq. (16).

References

Blanchard, Olivier, Diamond, Peter, 1994. Ranking, unemployment duration, and wages. Rev. Econ. Stud. 61, 417–434.

Chang, Yongsung, Gomes, Joao, Schorfheide, Frank, 2002. Learning-by-doing as a propagation mechanism. Am. Econ. Rev. 20 (5), 1498–1520.

Elsby, Mike, Hobijn, Bart, Sahin, Ayesigul, Valletta, Robert, 2011. The labor market in the great recession: an update. Brookings Papers on Economic Activity,pp. 353–384.

Eriksson, Stefan, 2006. Skill loss, ranking of job applicants and dynamics of unemployment. Ger. Econ. Rev. 7 (3), 265–296.

Eriksson, Stefan, Gottfries, Nils, 2005. Ranking of job applicants, on-the-job search, and persistent unemployment. Labour Econ. 12, 407–428.

Esteban-Pretel, Julen, 2005. The effects of the loss of skill on unemployment fluctuations. CIRJE Working Paper F-371.

Esteban-Pretel, Julen, Faraglia, Elisa, 2010. Monetary shocks in a model with skills loss. J. Money Credit Bank. 42 (7), 1235–1265.

Hall, Robert, 2005. Job loss, job finding and unemployment in the U.S. economy over the past fifty years. NBER Macroeconomics Annual,pp. 101–137.

Hosios, Arthur, 1990. On the efficiency of matching and related models of search and unemployment. Rev. Econ. Stud. 57 (2), 279–298.

Khalifa, Sherif, 2012. Job competition, crowding out, and unemployment fluctuations. Macroecon. Dyn. 16, 1–34.

Khalifa, Sherif, 2013. Labor mismatch, skill obsolescence, and unemployment persistence. J. Quant. Econ. 11 (1,2), 199–229.

Krause, Michael, Lubik, Thomas, 2004. On-the-job search and the cyclical dynamics of the labor market. European Central Bank Working Paper 779.

Ljunqvist, Lars, Sargent, Thomas, 1998. The European unemployment dilemma. J. Polit. Econ. 106 (3), 514–550.

Pissarides, Christopher, 1992. Loss of skill during unemployment and the persistence of employment shocks. Q. J. Econ. 107 (4), 1371–1391.

Pries, Michael, 2004. Persistence of employment fluctuations: a model of recurring job loss. Rev. Econ. Stud. 71 (1), 193–215.

Ravenna, Federico, Walsh, Carl, 2012. Screening and labor market flows in a model with heterogeneous workers. J. Money Credit Bank. 44 (2), 31–71.

Shimer, Robert, 2005. The cyclicality of hires, separations and job to job transitions. Fed. Reserv. Bank St. Louis Rev. 87 (4), 493–507.

Sims, Christopher, 2002. Solving linear rational expectations models. Comput. Econ. 20 (1–2), 1–20.